MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 104 No. 3

MEETING OF 1944 MAY 12

Professor E. A. Milne, President, in the Chair

The election by the Council of the following Fellows was duly confirmed:—

Jack Howard Clarke, B.Sc., Dominion Physical Laboratory, Wellington, New Zealand;

Alexander Kirkcaldy, 45 Upton Street, Launceston, Tasmania;

Charles Frederick Nelson Powell, B.Eng., 6 Lawn Road, Hampstead, London, N.W. 3; and

Howard Percy Robertson, B.S., M.S., Ph.D., Princeton University, N.J., U.S.A.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

William Douglas Anderson, Altair, 40 Pacific Street, Roslyn, Dunedin, N.W. 1, New Zealand (proposed by D. C. Berry);

Derek Martin Butcher, F.R.Met.Soc., 21 Hillcrest Road, London, W. 3 (proposed by J. F. Cox);

James Clendinning, O.B.E., B.Sc.(Eng.), A.M.Inst.C.E., 23 Westfield Park, Redland, Bristol, 6 (proposed by G. P. Lenox-Conyngham);

Henry Richard Didcock, 91 Hugh Street, Castleford, Yorkshire (proposed by F. J. Sellers); and

Robert S. J. Edwards, The Vicarage, Stanton, Shropshire (proposed by J. F. Cox).

Sixty-five presents were announced as having been received since the last ordinary meeting, including:—

Sir Arthur S. Eddington, *The Combination of Relativity Theory and Quantum Theory* (presented by the author);

About one hundred scientific books, formerly the property of the late Hon. Sir Lionel Guest (presented by his widow); and

Sir Harold Spencer Jones, *The Royal Observatory, Greenwich* (presented by the author).

THE SOLAR ECLIPSE OF 1936 JUNE 19

Notes on the Coronal Structure and the Relationships Observed between Chromospheric and Coronal Phenomena

S. Vsessviatsky in collaboration with E. Bougoslavsky

(Communicated by the Secretaries)

(Received 1944 March 31)

During the total solar eclipse of 1936 June 19, work with four standard coronographs located in the path of the eclipse at Beloretchenskaya, Ural, Omsk and Kuibyshevka yielded 30 excellent plates of the corona and chromosphere.*

A study of this material is of especial interest in that it allows conclusions to be drawn concerning the structure and changes in the structure of the corona, from records made during a two-hour period at intervals between the mean moments of observations shown in the following table:—

Station	Δt
	h m
Ural–Beloretchenskaya	0 11·0
Omsk–Ural	0 26·0
Kuibyshevka–Omsk	1 19·0

The plates were of excellent quality as regards freedom from background, sharpness, detail and contrast, fully revealing coronal formations and their structure.

A number of articles based on extended studies of these plates have already been published. Some of this material calls attention to many cases of changes observed in the inner corona and suggests a preliminary classification of different coronal formations.† Other articles were devoted to a discussion of the observed displacement of details in the outer corona from the point of view of the effect of rotation, and to an elucidation of the spatial disposition of some formations in the corona.‡

Photometry of the plates provided data for the evaluation of the law of diminution of the brightness of the corona, and likewise for the provision of isophotes.§

The final phase of the work as a whole contains a detailed account of the study of the structure of, and changes in, the inner corona. This work was finished in 1940, but publication was delayed.‖ The present article sets forth the results of this later investigation.

Method of Studying the Plates.—The large number of plates obtained permitted a study not only of the coronal formations themselves, from the edge of the chromosphere to the outer parts of the corona, but also of the changes which took place within the formations for the period of time covered. These studies were carried out on the basis of measurements made with the instruments of the Pulkova Observatory and of the Sternberg Astronomical Institute in Moscow.

Measurements of the displacement of coronal details on 23 pairs of plates were carried out on the blink-comparator, and the results were compared with those obtained from the plates of the chromosphere taken by the Pulkova expedition to Ak-Bulak.

* *Tr. I.A.U.*, 6, 377, 1939.

† S. Vsessviatsky, E. Bougoslavsky, A. Deutsh, *Reports of Soviet Expeditions*, 1, 15, 1939.

‡ S. Vsessviatsky, E. Bougoslavsky, *Reports of Soviet Expeditions*, 2, 5, 1941.

§ S. Vsessviatsky, V. Dombrovsky, *Reports of Soviet Expeditions*, 2, 104, 1941.

‖ S. Vsessviatsky, E. Bougoslavsky, *C.R.Ac.Sc.U.S.S.R.*, 25, 368, 1939.

Comparisons were also made with plates obtained with cameras of shorter focal length, and also with the original spectroheliograms received from Meudon and from the Mt. Wilson Observatory, which were taken in the period from the 12th and the 25th of June 1936, and which were received through the courtesy of Dr. Azambuja and Dr. Nicholson.

The Arch Systems of the Corona of 1936 *June* 19.—A characteristic peculiarity of the structure of the inner corona was afforded by the presence of systems of mutually

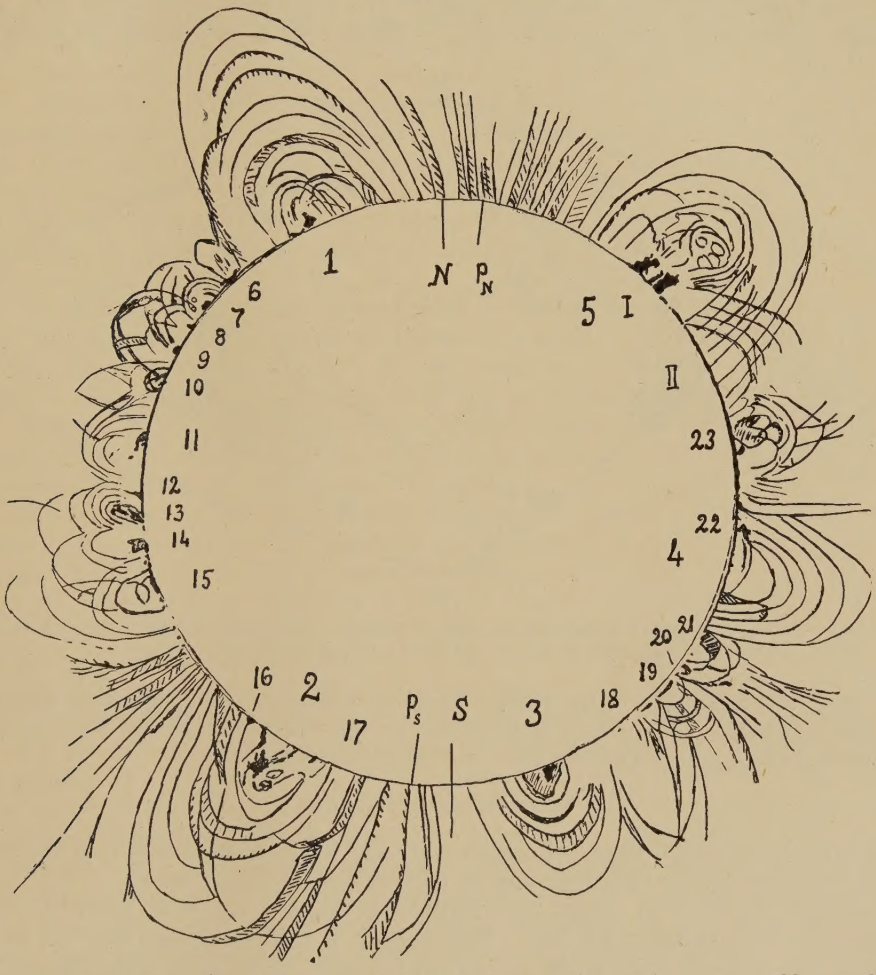


FIG. 1.—*Inner corona of* 1936 *June* 19. *Schematic drawing of coronal structure. Chromosphere from Ak-Bulak spectrograms.*

entwined coronal arches, each system having a practically common axis. These systems are assumed to have their origin in local centres of force on the surface of the Sun. During the eclipse of 1936 June 19 at least 20 such systems were observed, some of which contained more than seven arches. Internal to all of these systems, but not necessarily in the centre of each, were located protuberances or other equivalent chromospheric formations which unquestionably also arose from the same centre, or field of force, which was responsible for the given group of coronal arches. A schematic sketch of the inner corona of 1936 June 19 showing the coronal arch system is given in Fig. 1.

In most cases it was possible to determine with a fair degree of certainty the spatial distribution and structure of the coronal formations. The prominences intertwined

with their respective arch systems, and hence the systems themselves were juxtaposed with dark hair lines and flocculi. The positions of the latter were exactly determinable on the spectroheliograms.

Fig. 2 represents a freely drawn map of the surface of the Sun as reconstructed from spectroheliograms taken at the moment of the eclipse. The numbers refer to details corresponding to coronal formations on the limb during the eclipse. The central meridian corresponds to 6^h U.T.

It was found possible to take measurements of a large number of arches with great certainty. The arches were in the form of an ellipse with its centre near the surface of

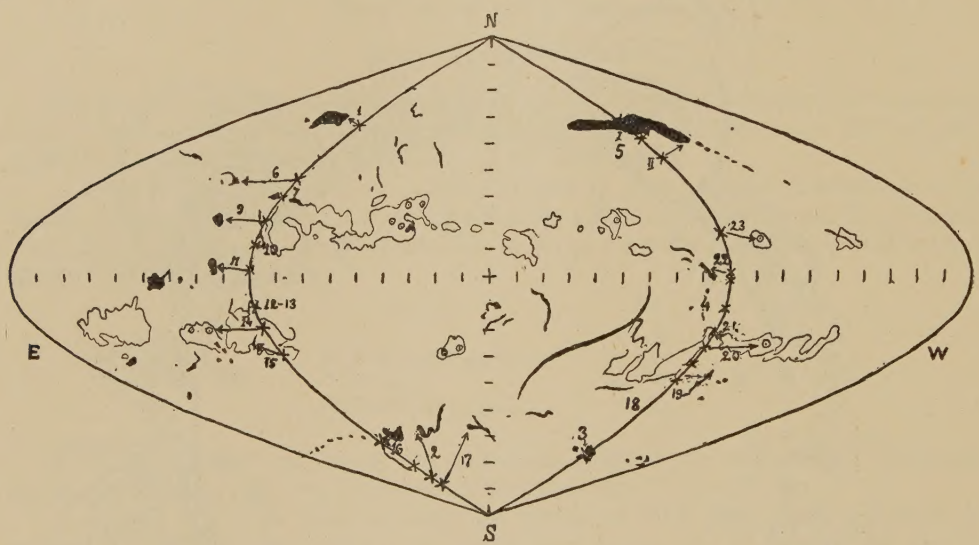


FIG. 2.—*Map of the Sun's surface for the time of eclipse. Apparent positions of the details of the corona in the limb* (×) *and their real positions on the surface.*

the Sun. The extensions of the arches of a given system increased regularly with their increase in height. The relation between the amount of eccentricity and the height was quite definite and agrees with the following empirical formula:—

$$(1-e)(\Delta+0{\cdot}15)=0{\cdot}091,$$

in which e is the eccentricity of the arch corrected for projection, and Δ is the height above the surface of the Sun, expressed as a fraction of the Sun's radius.

Velocities in the arch system as registered during the two-hour period were not great, and rarely reached 3 km./sec. Nevertheless, it was determined that the peak of the arch systematically rose at the rate of 0·4 km./sec. This result was obtained after correction for perspective and for the effect of the Sun's rotation. Arches which rose above the surface of the Sun became narrower or compressed.

Fans.—At higher heliographic latitudes, some of the more intensive systems of arches (Nos. 1, 2, 3 and 5 of Fig. 1) are associated with enormous coronal flows having extensions along the limb of several tens of degrees. To these we have given the name "fans". These fans represent grandiose coronal formations and mark the borders of enormous zones of excitation. A characteristic feature of the structure of a fan is the presence in its interior of a system of arches.

In the extremely high parts of a system of arches, the arches become a straight bundle of twisting, flowing, almost spoke-like beams, many of which may be followed for a distance of 3 or 4 solar radii. The sharp edges of the fan are curved and slope to conform to the arches of the internal system, and are deprived, it would seem, of the

summits of the external arches. The fans are readily distinguishable from each other and present a characteristic picture of the corona as the latter usually appears on an over-exposed photograph of the eclipse. In the intervals between the fans may be observed other characteristic coronal formations—systems of coronal rays.

Coronal Rays.—Fig. 1, and other photographs submitted, show the distribution of the coronal rays as of 1936 June 19. Of these, the most characteristic is the system of rays at the north polar region. A peculiarity of these rays is their tendency to distribute themselves as doublets. It was found possible to study the form of the rays in some detail. In most cases the rays were not straight, but were bent toward the adjoining fans, a fact which would indicate a correlation with disturbances in the region of the fans. Measurement of the displacement of the rays gives low but measurable values for a shift, which is satisfactorily explained by the revolution of the corona with the Sun. From this it is concluded that the system of rays may be considered to be a lattice occupying the regions between the fans. In many cases it was possible to follow the rays right up to the chromosphere, to points of origin which may be considered to be chromospheric swellings, the slope and distribution of which harmonized with the structure of the ray system.

The ray systems seem to be characteristic formations in the corona, and they correspond to "undisturbed" regions of the Sun's surface. The coronal rays of these systems, the "chromospheric rays", are fully distinguishable from the fine rays which terminate the fan-like forms, or helmet-shaped coronal effluvia, over the prominences (see detail 3, Fig. 1), and also from the straight-line effluvia not having a sharply defined outline, and which travel in a radial direction from the regions of flocculi and faculæ. The chromospheric rays characterize the structure of the internal corona, and their intensity falls rapidly, at approximately the same rate as the brightness of the inner corona. The fan-rays and the rays over the flocculi are a peculiarity of the external corona. The gradient of the decrease in brightness with distance is not steep. The structural peculiarities of these rays differ from those of the chromospheric rays—and it may be assumed that there is a corresponding difference in their physical nature.

Phenomena of the Chromosphere and in the Surrounding Regions of the Corona.—A detailed study was made of changes in the prominences. In many cases it was possible to establish the metamorphosis of the prominences into the coronal state. In the 2-hour period it was possible to observe the merging or "solution" of prominence condensations and splashes into the coronal clouds or envelopes. The more positive of these observations were made in region 2, in an area of eruptive prominences which disappeared at the moment of photographing at Kuibyshevka (see Fig. 3), and in region 5 over a great prominence. The rate of displacement of isolated masses moving along definite, stable lines toward the surface of the Sun in different parts of the prominences was determined, and it was found that there were sharp changes in this rate in a number of cases. Over separate parts of high prominences in the corona, it was possible to observe localized arch-clouds. This is an indication that some regions of the prominences represent a source of local field similar to those centres on the surface of the Sun which give rise to normal arch systems in the corona. In addition to being able to observe the merging of condensed parts of the prominences into coronal clouds, we were able, in some cases, to follow the flow of coronal material from the swellings of the prominences and from the chromospheric swellings. This is clearly visible in the neighbourhood of prominences Nos. 5 and 2, in the chromosphere of the polar regions, and in region 23.

Coronal Flows over the Sunspot Regions.—Three active regions of spot formation were disposed close to the limb on 1936 June 19. These regions in the corona corresponded to a special formation characterized by a complicated structure and exceptional intensity (regions 14, 20, and 23, Figs. 1 and 2). Due to their form, we have named them "bushes". There is little doubt that here, in a limited space, there takes place an outflow of considerable masses of the coronal matter—a phenomenon which is

characterized by an intense continuous spectrum. The pole of a region of "bushes" is evidently intensively and powerfully disturbed. This is shown by the complicated structure in comparison with other excited areas in the corona. Concerning these disturbed areas, another interesting fact is that on the borders of the "bush" areas in all three cases are to be found separate chromospheric splashes and eruptions of isolated chromospheric masses which travel at great speed with inclined trajectories. The structure of the chromosphere in the bush areas is characteristic of its disturbed state and fully corresponds to the structure of the coronal formations. The location and

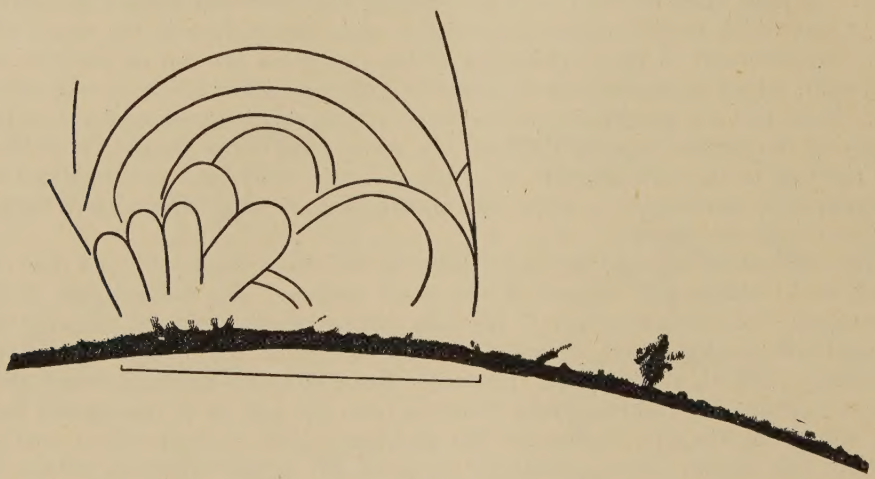


FIG. 3.—*Disturbed region of chromosphere of western coronal "bush" (region 23 of Fig. 1). Enlarged from Ak-Bulak spectrogram; Hα line.*

direction of the chromospheric swellings correspond to the details of the bushes (see Plates).

General Conclusions.—The general conclusions reached in our discussion may be summed up as follows:—

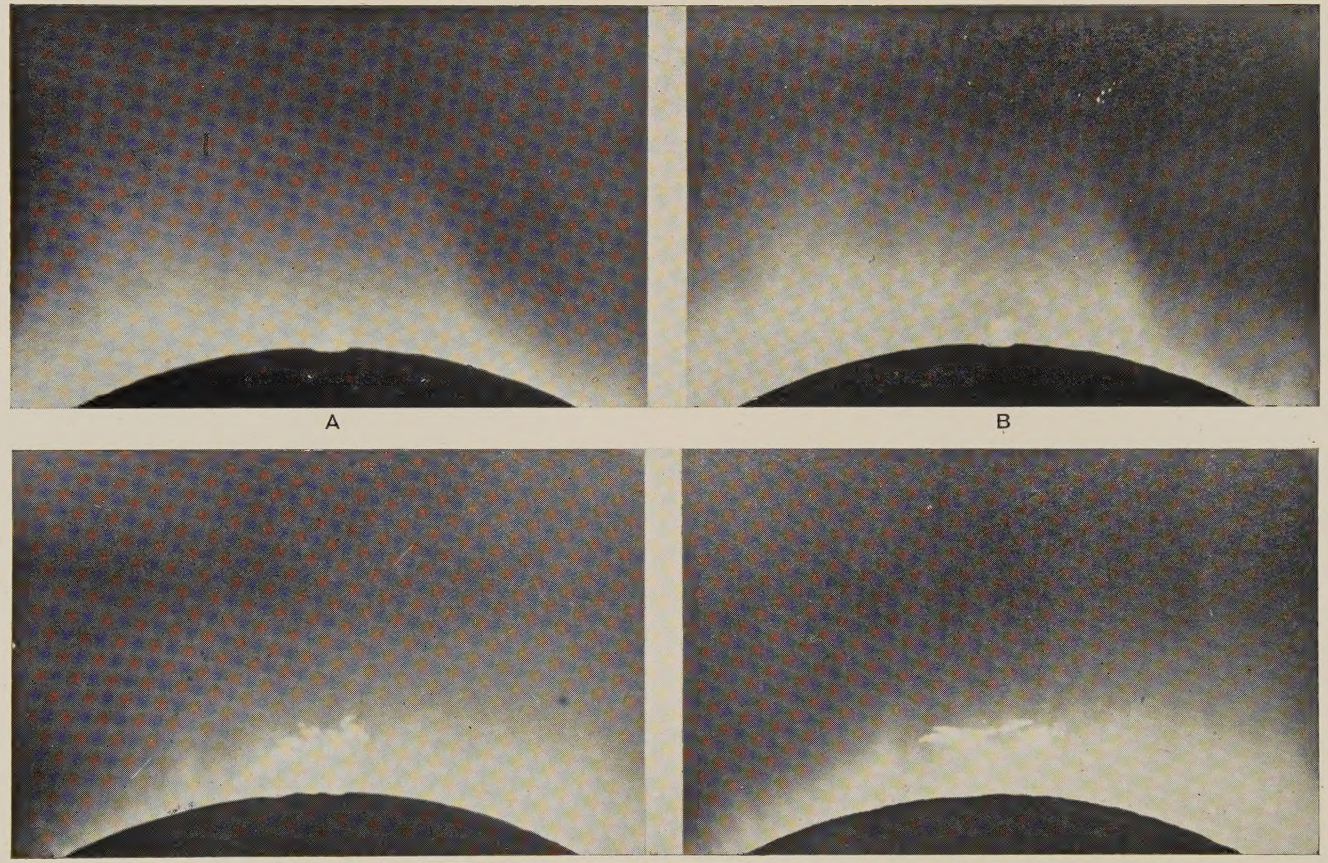
1. Phenomena in the chromosphere and in higher-lying coronal regions are due to foci or centres of excitation on the surface of the Sun. Their frequency and their peculiarities of distribution are related to the heliographic latitude, and they change with phases of the Sun's activity. The extent of the field of a disturbance centre in the sub-polar zones was, on the average, greater than in the equatorial zones, due to the significantly greater frequency of occurrence of active centres in the equatorial zones.

2. The coronal arch systems and the fan forms involved with them occasionally take on a helmet-shaped summit which changes into thin rays distributed over the centres of excitation, and these are connected by dark fibrous formations or dark flocculations. In the chromosphere, excitation gives rise to prominences which are imprisoned, so to speak, in the interior of the coronal arch systems. The external appearance of the corona, if judged by the peculiarity in the manner of distribution of the fans, is affected by eruptive processes on the surface of the Sun. At times of maximum solar activity, when the fibrous structures appear at high latitudes, the fans likewise respond to these high-latitude processes, and the corona takes on a more ragged, less compressed form than at times of minimum activity, when the dark fibrous areas are restricted to regions near the equator.

A study of the photographs of fourteen eclipses shows that the inclination of the axis of the fans to the solar surface changes in relation to changes in solar activity. The

Region of disappeared prominence (region 2 of Fig. 1) enlarged × 3·25 from Plates of Standard Coronographs.

A. Beloretchenskaya $4^h\ 00^m$ U.T. B. Ural $4^h\ 11^m$ U.T.
C. Omsk $4^h\ 37^m$ U.T. D. Kuibyshevka $5^h\ 56^m$ U.T.



A and B. "Normal" arch system (region 3 of Fig. 1).

A. Omsk $4^h\ 37^m$ U.T. B. Kuibyshevka $5^h\ 56^m$ U.T. (Enlarged × 3·25.)

C and D. Region of great protuberance (region 5 of Fig. 1).

C. Omsk $4^h\ 38^m$ U.T. D. Kuibyshevka $5^h\ 56^m$ U.T.

axes of the fans during maximum activity are practically normal to the surface, but as the activity becomes less the inclination increases, forming an angle of about 35° to the normal at a time of minimum activity.

3. The fan-shaped forms which characterize the structure of the corona over excited areas of the Sun's surface have a discrete structure and represent a system of successive arches. The helmet-shaped summits observable on many of the fans bear witness to the presence of vertical fields in different regions of the corona at considerable distances ($\frac{1}{2}$ to 2 radii) from the surface.

4. The structure of the corona over unexcited regions of the Sun is characterized by the presence of "chromospheric rays" formed in the space lattice of the effluvium. The position of the coronal rays coincides with local upheavals and minor eruptions in the chromosphere, as this phenomenon is observed on the limb, and likewise coincides with those lighter regions observed on the spectroheliogram showing the "quieter" spotted areas of the Sun.

5. The influence of the general field of the Sun is reflected first in the peculiarity of the inclination of the axes of the fans to the normal, and by the change in their inclination in relation to the phase of solar activity. The influence is shown secondly by the existence of a weakened luminosity inside the corona in a narrow equatorial belt having a width of about 7°. This is clearly visible not only on the photographs of the eclipse of 1936 June 19, but also on photographs of the internal corona and on isophotes of other eclipses.

Kiev Astronomical Observatory,
Sverdlovsk, U.S.S.R.:
1943.

PRELIMINARY RESULTS OF TESTS OF AND OBSERVATIONS WITH THE REVERSIBLE TRANSIT CIRCLE OF THE ROYAL OBSERVATORY, GREENWICH

Sir H. Spencer Jones, F.R.S., and R. T. Cullen

(Received 1944 May 10)

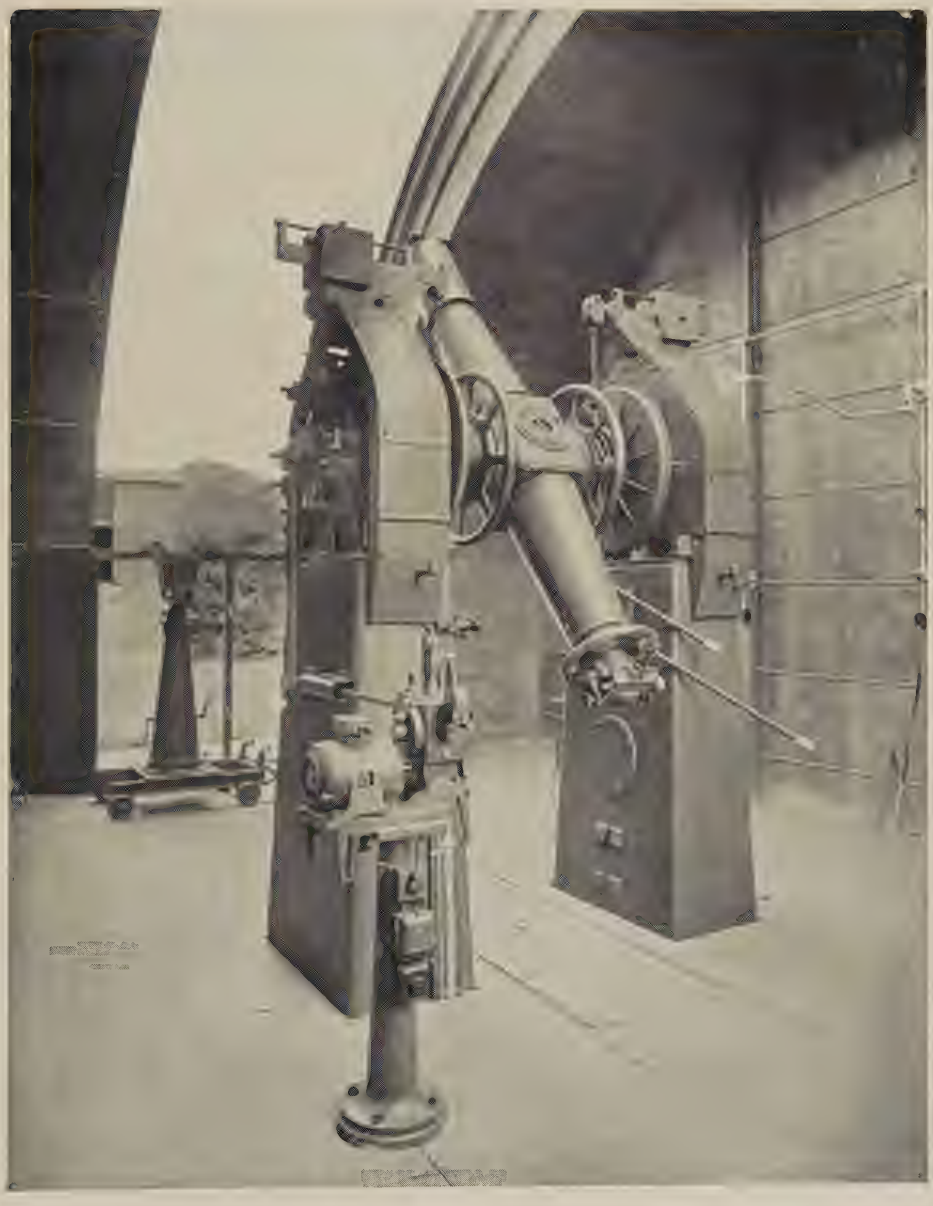
Summary

The principal features of the new reversible transit circle of the Royal Observatory, Greenwich, are described, and an account is given of the preliminary tests of the instrument, including the figure of the pivots and the construction of the micrometer eye-end. The diurnal and seasonal changes of the instrumental errors of adjustment have been investigated, and the importance of a close control over the error of collimation is emphasised. A correlation between the observed azimuth error and wind direction has been found. The investigations lead to some conclusion about the housing of meridian instruments.

With the installation of the reversible transit circle in 1936, a new era in meridian astronomy at the Royal Observatory, Greenwich, commenced. The transit circle, designed by Airy and installed in 1851, which defines the prime meridian of longitude, had remained the fundamental instrument for meridian observations since that date. No instrument in the history of astronomy has had such a record of work, and the value of the Greenwich meridian observations is due, in large measure, to the continuity of observation with the same instrument. The object-glass has been twice refigured to get rid of stains; although the surface is so badly stained that there is a considerable loss in transparency, the lenses have become too thin for a further refiguring to be contemplated. The graduated silver circle has become worn, through constant cleaning to remove the tarnishing produced by the polluted atmosphere of Greenwich, so that many of the divisions are only read with difficulty; some had completely disappeared and were recut at the Observatory. In addition, the instrument is non-reversible and is so housed that refraction anomalies are troublesome.

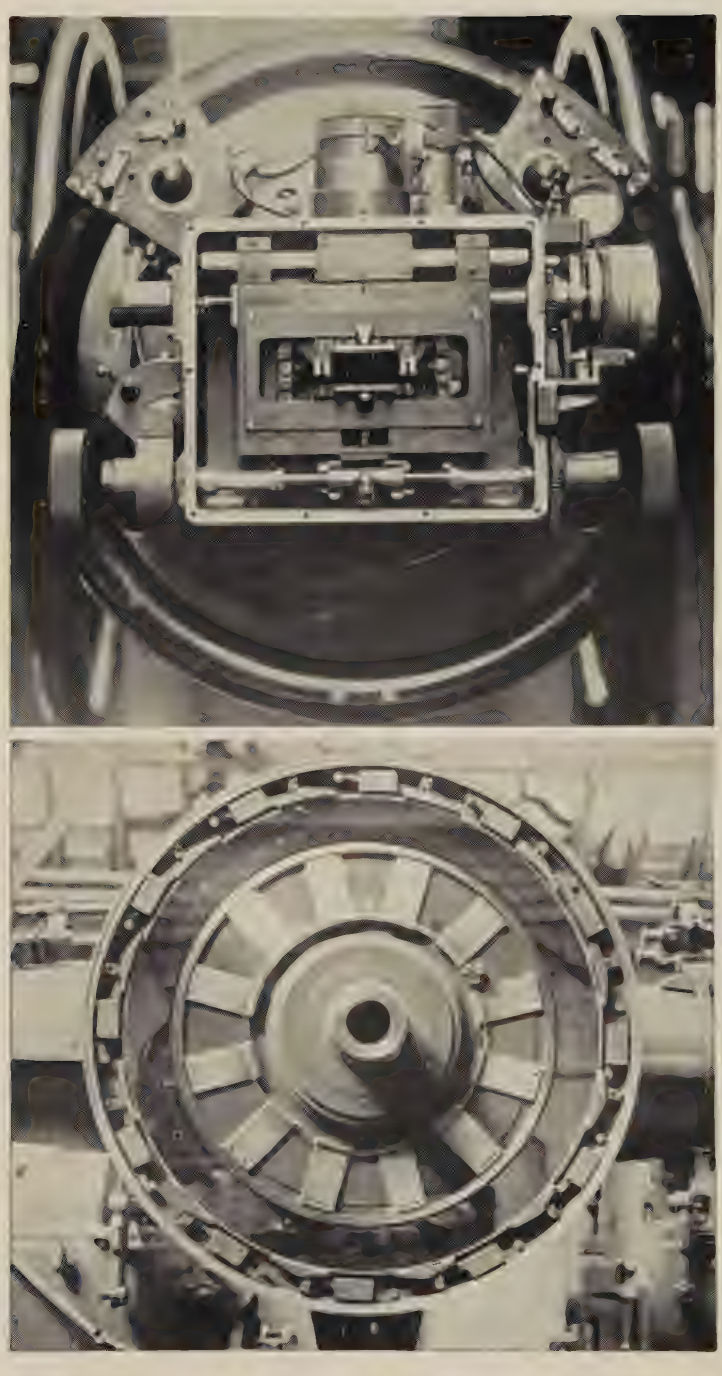
The new instrument, constructed by Messrs. Cooke, Troughton & Simms, Ltd., is a reversible instrument, which follows closely in design the reversible transit circle of the Cape Observatory, designed by Gill, of which a detailed description is given in the *History and Description of the Cape Observatory*. The objective has an aperture of 7 inches and a focal length of 102 inches. The travelling wire of the impersonal micrometer is driven by a synchronous motor, with a continuously variable gear, the drive passing up the outer face of one of the piers, through one of the pivots and then inside the tube to the eye-end. A reversing gear is fitted at the eye-end, together with a differential sun and planet gearing to enable the observer to maintain an accurate bisection of the star image during transit.

The principal feature in which the design differs from that of any previous instrument is the adoption of glass circles, with etched graduations, to eliminate the trouble of tarnishing to which graduations on silver are liable. It may be mentioned that in the Cape instrument the graduations of the fixed circle are made upon an inlaid band of iridio-platinum (30 per cent. iridium and 70 per cent. platinum), those of the movable



Reversible Transit Circle, Royal Observatory, Greenwich.

Sir H. Spencer Jones and R. T. Cullen, Preliminary Results of Tests of and Observations with the Reversible Transit Circle of the Royal Observatory, Greenwich.



A. Transit Micrometer, with cover plate removed.

B. Glass Circle, showing method of mounting and counterpoising.

circle being made upon an inlaid band of silver. The design and construction of the glass circles required considerable care. The circles are of 28 inches diameter, and the etched graduations are spaced at intervals of 5 minutes of arc around an annulus of 24 inches diameter. Each circle is supported on a spigot formed by 12 gunmetal arms, which are in turn attached to a cast-iron bracket fixed to the central casting. The spigot was turned in place after the arms were fitted. The coefficients of expansion of samples of the cast iron, gunmetal and glass were determined at the National Physical Laboratory; the dimensions of the supporting gunmetal arms were chosen so that the difference of expansion of the cast iron and glass is compensated. The rotation of the circle is prevented by a single stop-pin. The weight of each glass disk is approximately 16 lb. Flexure in the plane of the disk is minimised by a system of twelve counterpoises, bearing on the outer edge of the disk. Each counterpoise consists of a lever arm, weighted at one end and carrying at the other end a self-aligning arm; leather pads at the end of this arm bear on the edge of the circle. In any position of the circle the six lower counterpoises are in operation. The weight on each arm is adjusted so that the maximum pressure of each counterpoise is the same, and so that the total weight relieved is slightly less than the weight of the circle. Each circle is protected by a cover disk, so that only the graduated annulus is exposed. The etched graduations are sharp and excellent for setting upon.

The instrument is supported from two hollow cast-iron piers, which are mounted upon a strong cast-iron framework, which is adjustable for level. This framework is carried by a strong cast-iron base plate, which is adjustable in azimuth. Thus adjustments to the level and azimuth of the instrument can be made without upsetting the alignment of the bearings. The base plate is mounted in a hollow cast-iron tank, filled with Rubrex Oil No. 40 to prevent rusting of the screws which control the level and azimuth adjustments. The hollow piers are filled with a solution of ethylene glycol in water, containing 25 per cent. by volume of ethylene glycol. The freezing-point of this mixture is $-12°{\cdot}5$ C. $7\frac{1}{2}$ cwt. of ethylene glycol were used.

As originally constructed, the weight of the instrument was relieved from the pivots by means of springs and rods acting through self-aligning swivel stirrups, which pressed upwards on to the axis, ball journal races making line contact with a fitting on each axis to reduce friction. The springs were adjusted, calibrated, and fitted with a scale and pointer to indicate the amount of weight relieved. Both supporting rods and springs were carried from the azimuth adjustment plate. Tests on the instrument, mounted at the contractor's works, showed that the readings given by the instrument were not consistent. When, for instance, the instrument was set on the nadir and the microscopes were read, then on rotating the instrument through 360° and again setting on the nadir, a different reading was obtained on the microscopes. The readings varied, moreover, with the direction in which the instrument was rotated. The cause of the inconsistencies was either torque in the axis casting or constraint exerted by the weight-relieving device. The results became more capricious as the weight on each bearing was reduced, pointing to the counterpoise gear as the cause of the trouble. Some modifications to the gear were made, which reduced, but did not entirely eliminate, the inconsistencies; these still amounted to one or two seconds of arc with a weight of 50 lb. on each pivot. This method of relieving the weight from the bearings was therefore abandoned. A new counterpoise system was made in which the instrument is supported within anti-friction collars by two chains, attached to the ends of counterpoised lever arms. Each counterpoise consists of a large fixed weight and a smaller adjustable weight, which can slide along a calibrated arm; by the adjustment of this weight, the weight on the corresponding bearing can be varied within a range from 0 to 50 lb. The weight-relieving gear is supported from the piers, below the block which carries the micrometers for reading the circle, so that there is no tendency for these blocks to be strained. With this self-equilibrating method of relieving the weight from the pivots,

the inconsistencies in reading are no longer obtained. The instrument is used with a weight of 30 lb. on each bearing.

The instrument was erected at the Royal Observatory in February and March 1936. Researches of various kinds were initiated, concurrently with observations for the establishment of a fundamental system of right ascensions. These were continued until the air attacks on London in the autumn of 1940. The objectives of the instrument and of the two collimators and the micrometer eye-end were then dismounted and removed to a place of safety. Enough material has been obtained and conclusions formed to guide the planning of post-war observations with the instrument, and the performance of the instrument has been improved. The investigations have revealed various ways in which systematic discordances of instrumental origin can enter into meridian observations, and the results may be helpful to others in possession of, or planning the design of, similar equipment.

The transit pavilion is semi-cylindrical in shape, the axis of the cylinder coinciding with that of the instrument. The interior diameter of the cylinder is 30 feet. The internal width of the pavilion is 20 feet. Two shutters, of semi-cylindrical shape, can slide back in the E.–W. direction to give an aperture of 8 feet along the meridian. Thermal insulation of the pavilion is provided by slabs of granulated cork, 3 inches in thickness. The housing is covered externally with copper sheets, lapped together. The reversing carriage, when placed in position under the instrument and raised, lifts the instrument from its bearings. The carriage is then run along steel rails to the north of the piers and the instrument is turned through 180°, after which the carriage is run back and the instrument is lowered into its bearings. A platform is fitted to the carriage, which is employed for making observations on the nadir. When not in use the carriage is run to a position at the side of the pavilion to the west of the turntable, a turntable being provided for this purpose.

The foundation which carries the instrument is of concrete, extending to a depth of 9 feet and weighing 75 tons. It had been intended to carry the foundations to a depth of 15 feet, but the subsoil of hard compacted gravel was so firm that 9 feet was judged sufficient.

The pavilion is surrounded by a surfacing of concrete, measuring 50 feet from north to south and 55 feet from east to west, to carry rain-water away from the foundations. About 4 feet from the edge of this surround is an isolation trench, 3 feet deep, to keep the upper portion of the foundations dry; this trench is covered with slabs of reinforced concrete $2\frac{1}{2}$ inches in thickness.

The collimators, which are of the same aperture and focal length as the transit instrument, are mounted on substantial foundations, inside brick houses, roofed with concrete, to the north and south of the instrument. The distance between the centre of the instrument and the centre of each collimator is 50 feet.

One revolution of the R.A. micrometer screw is equivalent to $55''{\cdot}7$ or $3^{s}{\cdot}7$ for an equatorial star. The R.A. micrometer carries with it a metal contact wheel, into which five studs are inserted to break the contact. One of these studs corresponds to zero reading of the micrometer head. The motion of the travelling wire is very uniform; in order to prevent an inaccurate bisection of the star image being maintained throughout a transit it was decided, after some experience had been gained, that five successive bisections on this stud should be retained for a normal transit observation. The moving wire is deliberately removed from the star after each bisection, so that the observation of a transit is based on five completely independent bisections, whose mean time corresponds to the time of the centre one. A loud speaker has been installed to give audible signals when the studs break the contact. The five studs on the wheel are not equally spaced; the observer knows by the rhythm of the pips from the loud speaker the correct stud at which the bisection must be made, and is able to concentrate on an accurate bisection of the star image just before the appropriate instants. This relieves the

observer of the necessity of maintaining bisection throughout a transit and results in increased accuracy. It is found that the probable error of a single transit is less than one-hundredth of a second of time.

The detailed investigation of the equipment and the results derived from observational data will first be described. The effect on the observations of the buildings housing the equipment will then be discussed.

Instrumental Errors

These may be divided into two distinct groups:

I. Errors that are constant or nearly so.
II. Those which are variable with diurnal and seasonal fluctuations in temperature.

The present investigation is concerned primarily with right ascension observations.

The errors in group I comprise:

(*a*) Errors due to inequalities of the pivots.
(*b*) Errors produced by a faulty fixture of the frame carrying the moving wire, called "frame correction".
(*c*) Errors due to the moving wire lagging behind the micrometer screw reading when in motion, termed "lag".
(*d*) Division errors of the circle graduations.

Those in group II comprise:

(*e*) Errors of collimation.
(*f*) Errors of level.
(*g*) Errors of azimuth.

The errors in group I can be determined and allowed for; when possible, it is preferable to reduce them to negligible quantities.

(*a*) *Pivot Errors.*—Errors due to irregularities of the pivots should be determined with extreme accuracy, as such errors are not eliminated by reversal of the instrument, except for stars in the zenith. Pivot errors affect the determination of collimation and the times of transit of stars across the meridian. It is desirable that they should be sufficiently small in amount for the effect on collimation to be insensible.

The pivot errors are determined by the method introduced by Airy in the Greenwich transit circle. In this method the horizontal axis of the telescope is converted into a horizontal collimator. An object-glass, mounted into a slightly cone-shaped tube, fits into a correspondingly conical hole in one pivot. Into a similar hole in the other pivot is fitted an apparatus carrying a fine mark, with provision for adjustments for focus and centring. The object-glass and mark can be interchanged in the two pivots. The mark is focused in the principal focus of the object-glass in the other pivot and viewed by a telescope, provided with micrometer eyepiece, mounted on the external face of the pier with its axis parallel to and nearly coincident with the axis of the transit circle. The displacements of the mark as the instrument is rotated are measured. Thus the combination of the effects of the inequalities of the two pivots, with which alone we are concerned, is directly measured.

When the telescope was first erected, preliminary tests of the pivots, whose diameter is 4·6 inches, were made with the pivot-testing telescope. These showed that one or both pivots were definitely oval. Determinations of collimation were made by three methods: (1) by observing north and south collimators, without reversal of the telescope; (2) by observing north or south collimator and reversing the telescope; (3) by observing the nadir and reversing the telescope. The determinations by the first two methods were in agreement; those by the third method showed a systematic difference of about

0″·6. The effect of the observed pivot errors on the determination of the line of collimation by methods (1) and (2) were negligible; they accounted for not more than 15 per cent. of the discordance by method (3). This indicated a difference in diameter of the two pivots of 0·0003 inch. The pivots had been tested at the works by ring tests; it was therefore probable that the smaller pivot was the more oval, as ring tests can only compare the greatest diameters of the two pivots. It was decided that the pivots should be lapped *in situ*, with the telescope mounted on the reversing carriage, by a representative of the contractors, starting with the pivot of larger diameter.

The pivot-testing telescope was not suitable for rapid testing of the figure of the pivots during lapping. To mount the equipment and obtain the observations takes a considerable time, and no information is provided about the figure of each pivot separately. A Krupp mikrotast gauge was therefore obtained to enable the figure of the pivots to be rapidly tested. This gauge consists of a pair of plane jaws inclined to one another at an angle of 39°, between which is mounted a movable foot. The displacements of the foot are magnified mechanically and read from a circular dial to an accuracy of a micron by direct reading, or to a decimal of a micron by estimation. The difference of the readings obtained with the mikrotast gauge, when placed on two truly cylindrical pivots of slightly different diameter, gives at once the difference in diameter of the pivots, because the cosecant of $19\frac{1}{2}°$, half of the angle between the jaws, has the value 3. When the gauge is moved round a pivot which is not truly cylindrical, the errors of figure are shown by the variations in the reading of the dial. The reading for any position of the gauge represents a combination of the errors of figure at three points: the two points where the jaws touch the pivot, and the intermediate point where the movable foot is in contact with the pivot. It can be shown that with the commonest types of error of figure, elliptical or pseudo-cylindrical, the mikrotast readings determine the error of figure with a magnification somewhat greater than 3.

The first tests with the mikrotast gauge were made after some preliminary lapping of the pivots with emery and rouge, and showed a variation in the readings on each pivot of about 4 microns and a difference in diameter of about 2 microns. The gauge indicated the portions of the pivots where lapping was chiefly required and the figure of each pivot was rapidly improved. As the accuracy of figure was improved, it was found necessary to allow the pivots to stand for at least an hour to equalise their temperature before tests were made. Readings were taken at intervals of 10° round the pivots. The figure was considered satisfactory when the variations in reading on each pivot showed no departure at any point from the mean exceeding 0·25 μ; the difference in the mean diameters of the two pivots was 0·28 μ.

Observations were then made with the pivot-testing telescope equipment in order to obtain the relative pivot errors. The errors so determined were larger than could be accepted as correct, in view of the consistency of the readings obtained with the mikrotast gauge. It was noted that the definition of the dot was not the same for all positions of the transit instrument, suggesting an aberrational effect such as might be introduced by the tilt of a lens. Repeat series were therefore made, turning the collimating lens into different positions, but keeping the pivot-testing telescope in the same position; then further series, keeping the collimating lens in the same position but turning the pivot-testing telescope into different positions. The apparent errors were altered by the first change but not the second, pointing to the collimating lens as the cause of the discordance. Investigation showed that its axis is not coincident with the axis of the pivots; it is probable that the conical holes in the pivots are not exactly coaxial with the pivots. Taking observations in four positions of the collimating lens at 90° apart, the true pivot errors could be separated from the spurious errors introduced by the tilt of the lens. The spurious errors were in the form of a double harmonic wave with a total amplitude of about 0″·5 in each co-ordinate, corresponding in linear measure to 3 microns. When the spurious errors were allowed for, it was found that

true pivot errors were appreciably less than one-tenth of a second of arc, in agreement with the conclusions derived from the mikrotast gauge observations.

(*b*) *Frame Correction.*—This error was detected by setting the R.A. moving wire on one of the fixed wires; it was found that the micrometer readings for the position of coincidence varied with the position of the telescope. In other words, the reading corresponding to the line of collimation varied with the circle reading or declination. Observations were also made by setting the movable wire on a speck of dust on the horizontal wire, with similar results. The effect was found to be greatest at one end of the field, diminishing to a minimum at the other end. The fixed wires are mounted on an immovable frame, and the existence of the effect therefore indicated a looseness of the frame carrying the moving wire.

Similar variations in reading depending upon the position of the telescope were obtained when the Z.D. wire was set on a fine speck of dust on a fixed vertical wire and

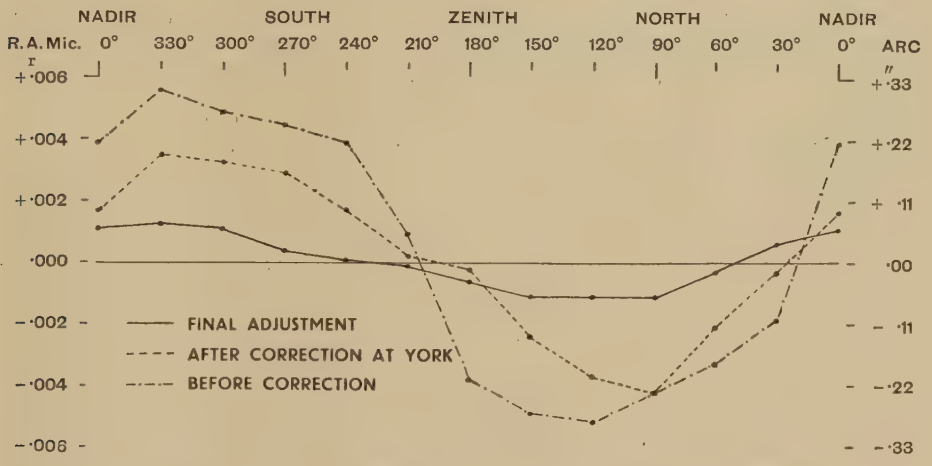


FIG. 1.—*Frame correction for position of micrometer slide showing maximum effect.*

the reading of the Z.D. micrometer was taken. The maximum displacements were of the order of 0″·6 for both the R.A. and the Z.D. movable wires.

The motion of the R.A. slide is guided by a guide rod of rustless steel, 18 cm. in length and 1 cm. in diameter. The central portion of the rod fits into a V-shaped groove, $5\frac{1}{2}$ cm. in length, in a cast-iron block, forming a part of the micrometer box casting; it is clamped into position by a flat plate, which bears on a flat on the central portion of the rod. The R.A. slide is guided by two bearings, 2 cm. in width; the portion of each bearing, which forms an integral part of the slide, is partially cleared away, to leave two segments, each of about 60° angle, bearing on the rod. The upper part of the bearing is clamped to this by two screws, and bears along a segment of about 120° angle.

The micrometer eye-end was returned to York for correction of this trouble and also of the trouble from lag of the moving wire, referred to below. After return from York, it was found that though the effect had been reduced in magnitude, it still persisted and showed the same general features. Much time was spent in investigating the cause of the trouble. It was finally reduced to negligible amount by a careful adjustment of the screws clamping the guide rod to the main casting and of the screws holding the bearing surfaces of the wire frame against the guide rod. In making these adjustments the slide was moved backwards and forwards to test the ease of motion and the freedom from constraint. The final adjustment has been tested at intervals and has been found to remain satisfactorily constant.

The diagram shows the nature of this error at different pointings of the telescope, for the position of the slide at which the effect was a maximum; the observed effects when first detected, after the eye-end had been returned from York, and after final adjustment at Greenwich, are shown separately. The plottings are micrometer readings in the sense mean *minus* observed values, for the circle east position of the instrument. At the centre of the field, where most transits are observed, the corrections are smaller and are negligible in amount.

Similar adjustments were made to the Z.D. micrometer frame. It was not found possible to reduce the range of the effect below about $0''{\cdot}2$, the extreme values being found when the telescope is in the north and south horizontal positions. It may be noted that an undetected discordance of this nature would enter directly into the determination of horizontal flexure and would give a spurious result for the flexure.

(*c*) *Lag of Travelling Wire.*—Preliminary observations of right ascension were compared with the R.A.'s given in the FK3 fundamental catalogue. It was found that the stars near the pole, whether observed above or below pole, appeared to transit too early as compared with equatorial stars. The order of magnitude of the discordance was about half a second of time at 7° from the pole. The discordances were the same whether the observations were made with the moving wire hand driven or motor driven, and were the same in the two positions of the instrument.

Observations of right ascension were then made by tapping transits across the fixed wires. The R.A.'s so obtained showed no appreciable deviations from the catalogue places. This definitely located the trouble in the micrometer. The discordance could be explained by supposing that the moving wire lagged behind the screw when the micrometer is in motion by about one second of arc, or 12 microns.

There are two methods of observing close polar stars: (*a*) by a normal transit, using the slowest possible motor drives; (*b*) by setting the motor to equatorial speed, running the wire alternatively backwards and forwards and tapping by hand the instants of its transit over the star. If there were a lag in the wire, the two methods would not give the same result and the difference could be used to infer the amount of lag. On observing by the two methods a selection of circumpolar stars, a difference between the R.A.'s obtained by the two methods was revealed, amounting to $0^{s}{\cdot}44$ at $6\frac{1}{2}°$ from the pole. This gives $0''{\cdot}7$ as the lag of the wire behind the screw reading.

The lower bearing of the micrometer slide consists of a slot in which runs a tail-piece on the slide. It appeared possible that this exerted a frictional constraint, tending to twist the moving frame when the micrometer is in motion.

When the micrometer eye-end was tested at York, it was set up with the Z.D. head at the top and the slides vertical. A sensitive bubble was attached to the main slides; it was found that the bubble would travel undisturbed in one direction, but on reversal it ran through $20''$ and showed a tendency gradually to correct itself. On removal of the main lead-screw, the motion of the slide in this direction was found to be uneven and harsh. This was corrected by easing the slide which guides the tail-piece. The guiding rod for the R.A. slide was found to be slightly convex to the eyepiece and the Z.D. slide showed a tendency to jump on reversal of the motion. These faults were attributed to settling down of the metal parts after machining and were cured by the removal of very small amounts of metal. After return of the micrometer eye-end from York, further tests showed that the error of lag had been eliminated.

The absence of lag is now checked at intervals by the following purely instrumental method. The moving wire is motor driven backwards and forwards at equatorial speed and the instants of transit across a fixed wire are tapped off. The same is done with the wire driven at a slow speed corresponding to that of a star with high declination. These tests have shown that the lag has remained negligibly small.

(*d*) *Division Errors of the Fixed Circle.*—The determination of the error of every division is essential for accuracy. The method used and an analysis of the results will

be given in a further communication. It is sufficient here to mention that the determination involved over 200,000 microscope pointings. The most noticeable feature is a cyclic error, recurrent every $2\frac{1}{2}°$, ranging from $+0''{\cdot}19$ to $-0''{\cdot}18$. Superposed on this cyclic error are periodic errors with periodicities of 15°, 30° and 60°, and semi-amplitudes of $0''{\cdot}022$, $0''{\cdot}023$ and $0''{\cdot}077$ respectively. The total range in the division errors is about $0''{\cdot}7$.

Coming now to the second group of errors, the instrumental errors of collimation, level and azimuth are subject to both rapid and slow variations. It should be noted that with an instrument in which the level observation involves the use of the R.A. micrometer, an error in the assumed value of the collimation enters directly into the error of level and further produces an error in azimuth. Also, if azimuth marks are used, an error in the assumed value of the collimation will also enter into the determination of the azimuth of the instrument relative to the marks.

(*e*) *The Error of Collimation.*—The usual procedure in meridian observations is to determine the error of collimation at most once daily and to assume that the error varies very slowly. The preliminary observations of transits showed that the instrument was capable of very high precision, and it became clear that more attention must be given to the determination of instrumental errors, especially of the error of collimation. The observations were reduced in the first instance assuming a constant value of the collimation error throughout the day, and interpolating the level error from a smoothed curve drawn to represent the level errors determined from the nadir observations. It was found that, in general, the observations gave a spurious drift in clock error in the course of a night's observations. This appeared to indicate that an apparent drift in level, possibly arising from a drift in the collimation error, was superimposed on the real variations in level.

In the summer of 1938 intensive observations of both collimation and level errors, together with transit observations of clock and azimuth stars, were commenced. Starting at 9^h in the morning, continuous observations were made, when conditions were favourable, throughout the day and into the night, with the exception of about one hour on each side of apparent noon, when the Sun shines directly on the telescope. Marked diurnal changes of the error of collimation, which often exceed $0''{\cdot}5$, were soon apparent. These changes were associated with temperature variations. Thermometers were later introduced and readings of the temperatures in the dome, inside and outside the eye-end, and inside the tube at the object-glass end, were made at the time of each observation of the collimation error.

The variations in the collimation error are of two types. The first observation of collimation in the morning is taken under what may be termed steady temperature conditions, when the instrument has been left with the dome closed for several hours. Under these conditions the collimation error correlates closely with temperature, showing satisfactory stability and a slow annual variation with temperature.

Plotting the readings of the micrometer screw for the position of collimation against temperature, it is found that the reading decreases by 0·00022 revolution of the micrometer screw for each 1° F. rise in temperature. The micrometer screw has 36 threads to the inch. The screw is of mild steel and bears at one end against a diamond bearing, mounted on the cast-iron micrometer box. The distance from the end bearing to the centre of the field is 3·6 inches. As the micrometer readings increase when the wire moves towards the end bearing, the collimation changes can be accounted for on the assumption that the coefficient of expansion of the steel micrometer screw is 3×10^{-6} per degree C. greater than that of the cast-iron box. The amount is of the right order of magnitude, though somewhat larger than would be expected. Samples of the steel and cast iron are not available for measurement of the coefficients of expansion.

When the observations are continued throughout the day, however, under conditions

of changing temperature, the collimation reading changes by amounts that are too great to be accounted for by expansion effects within the micrometer box. The changes correlate *qualitatively* with the temperature differences outside and inside the micrometer, with the differences between the dome and the eye-end, and with the differences between the dome and the objective. These various temperature differences usually correlate very closely with one another, though they differ in range, so that it is not possible to associate the changes in collimation reading with any particular set of temperature changes. The correlation between changes in collimation reading and the various temperature differences is not, however, quantitative; similar temperature differences may be obtained on two days, but the variations in collimation reading may be much smaller on one day than on the other.

The sequence of temperature changes usually observed is as follows: in the morning, when observations normally commence, the temperature in the dome rises more rapidly than the temperature within the tube at either end; the differences become greatest near the time of maximum dome temperature. As the dome temperature falls, the other temperatures lag behind, so that the dome in the evening falls to a lower temperature than the interior of the tube. The temperature differences usually reach a minimum value at some time in the evening, though the dome temperature is still falling, and then begin to increase again.

As an example of the changes that occur, the collimation and level readings for 1940 May 13–14 are shown in Fig. 2, together with the differences of temperature between the outside and inside of the micrometer box. This day has been selected as one on which the true variations in level were very small. The true level error is the difference between the collimation and level readings; it will be noticed that the level readings closely reproduce the fluctuations in the collimation reading. Thus undetected fluctuations in collimation would be thrown into the level readings and spurious corrections for level would be adopted.

Generally speaking, large temperature changes correspond to large collimation changes and conversely. The changes in collimation are thus appreciably greater in the summer months than in the winter months. The correlation is not, however, sufficiently close for the collimation changes to be inferred from the temperature changes and the time devoted to the collimation observations to be thereby reduced.

During the course of a day the observed range in collimation may amount to as much as 0·018 rev., corresponding to 5×10^{-4} inches. Such a change is much too great to attribute to differential thermal expansions in the micrometer box. There must be some other cause which produces a relative change between objective and eye-end. Displacement of the objective itself cannot occur; the objective is held against two fixed stops, 120° apart and symmetrical with respect to the meridian, by a small spring-loaded plunger in the meridian, the spring being of sufficient strength to keep the objective against the fixed stops when its weight is acting against the spring.

The manner in which displacements of the eye-end relative to the objective, of the order of magnitude observed, might most readily be produced seems to be through temperature differences between some parts of the telescope tube and other parts. The design of the telescope is intended to prevent such differences of temperature occurring. The tube is in the form of two truncated cones of cast iron, each about 38 inches in length, with a central cube of 20 inches side. The cube has short flanged cylinders, forming the means of attachment for the truncated cones. Each of the truncated cones is surrounded by a conical copper sheathing, attached at the base of the cone, where it is fixed to the central cube, and free at the other end, so that no strain can be introduced by the different thermal expansions of iron and copper. Between the copper sheathing and the tube is an air space of about one inch. The purpose of the sheathing is to protect the tube itself from local heating and to prevent differences of temperature in different parts of the tube being created.

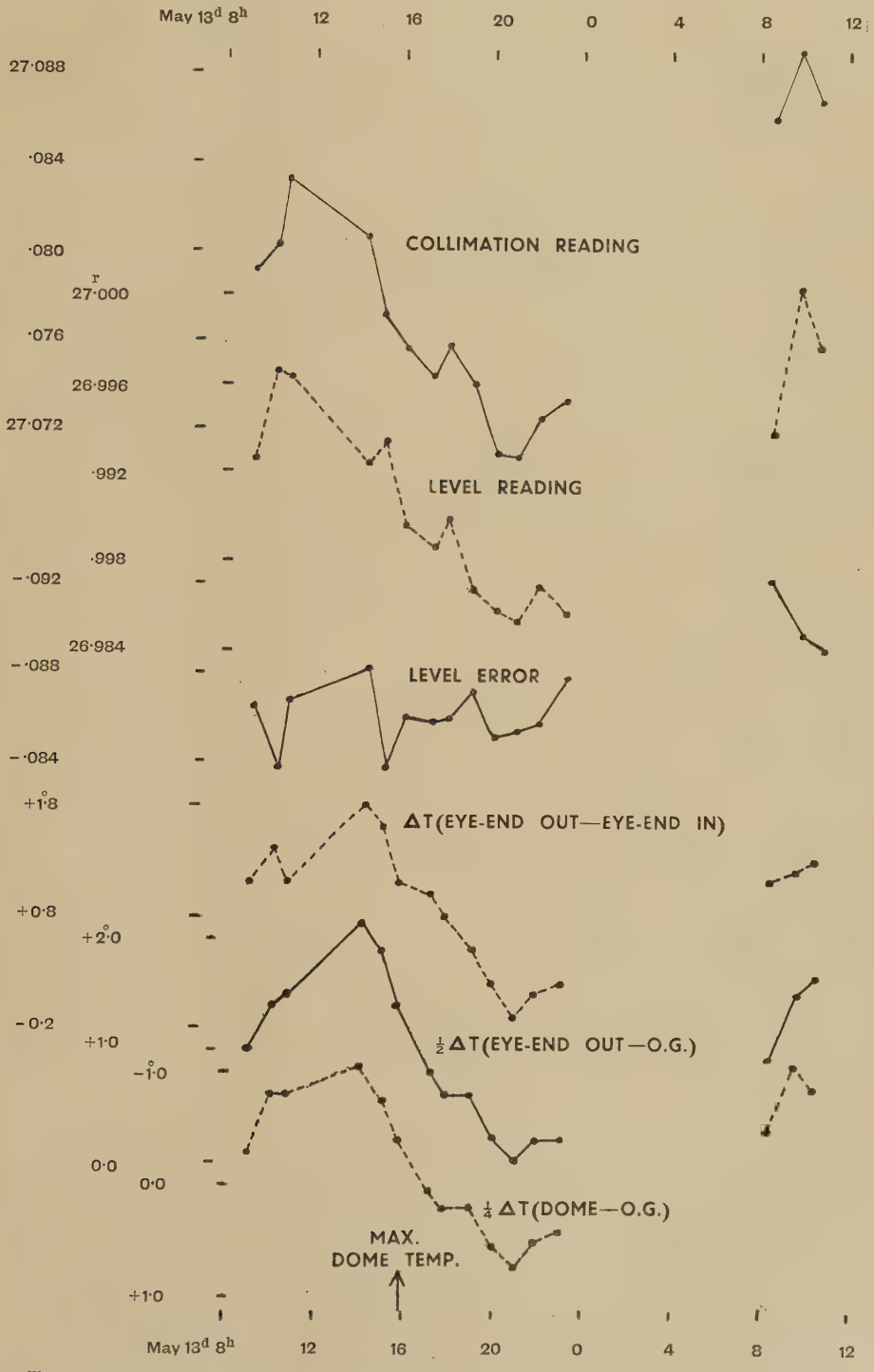


FIG. 2.—*The diagrams give the observed collimation and level readings, the inferred level error; the differences of temperature outside and inside the eye-end, between the eye-end (outside) and the object-glass (half scale) and between the dome and object-glass (quarter scale).*

But if, in spite of the copper sheathing, some parts of the tube could be at a different temperature from others by amounts depending on recent history, as well as on immediately prevailing conditions, displacements that would account for changes in collimation reading, of the order of magnitude that are actually observed, might occur. At the same time, though the changes would be expected to show some qualitative connection with temperature inequalities, they would not repeat exactly on different occasions. If, for instance, a temperature gradient across the tube were to exist, it can easily be shown that a difference in temperature between the east and west sides of the tube over the lower half of its length of a few tenths of a degree Centigrade, causing a slight bending of one-half of the tube, would be sufficient to account for the maximum observed changes of collimation reading. Variable differences in temperature between the two piers of the order of 1° C. have frequently been observed in the summer months. It might be supposed that the side of the tube adjacent to the warmer pier might be at a somewhat higher temperature than the other side, and that the difference in temperature between the two sides would be greater for the portion of the tube carrying the eye-end than for that carrying the objective; for the objective-end during star observations is at a higher level than the eye-end and therefore less likely to be affected by the piers. An equal bending of the two halves would not affect the collimation reading; it is only a differential bending that would produce any effect. If this were the cause of the diurnal changes in collimation, the effect on the collimation reading produced by a given difference of temperature between the two piers would be reversed by reversing the instrument. But such a reversal in the effect is not found; the general changes in collimation reading are qualitatively the same whether the instrument is in the position clamp east or the position clamp west. The changes are therefore not related to any tendency of the west pier to be systematically at a higher or lower temperature than the east pier.

The cause of the variations in collimation reading under conditions of changing temperature remain for the present obscure, and further investigations will be required, when the instrument is brought into use again, in order to find the cause. Unless it proves possible to trace the variations in collimation to their origin and to eliminate them, frequent observations of collimation will be needed in order that the instrumental errors can be sufficiently controlled to give results of the highest precision. This will necessarily slow down the observing appreciably, because frequent determinations of collimation consume considerable time.

(*f*) *Level Error.*—The daily variations in level at times exceed 1″·0. They are caused by variations in temperature affecting the piers, and possibly also the pivots, unequally. The direction and velocity of the wind are important factors in producing rapid changes of level. When the dome is open the wind, unless directly from the north or south direction, will blow on one pier more than the other, causing a fall in temperature of that pier. During the daytime the varying position of the Sun is also an important factor.

When the instrument was first brought into use, the variations in level were considerable and sometimes very rapid. This was particularly so on clear nights in the winter with a wind from a north-easterly direction. The west pier would then cool much more rapidly than the sheltered east pier and there would be a quick run off in the level.

A number of thermometers were therefore introduced, some of which were placed in direct contact with the metal surface of the piers. On each pier one thermometer was mounted about a foot below the pivots and another about two feet above the floor. A thermometer was placed in the oil in the base trough, and two others were mounted to give the air temperature on the piers at each end of the axis of the instrument.

It soon became apparent that the changes in level were closely correlated with the differences in temperature between the two piers. It was found possible to express the level error, with a probable error of about 0″·1, by the following formula:—

$$\text{Level error} = A + BT + C(T_E - T_W) + D(T'_E - T'_W),$$

where T is the temperature of the oil in the trough;

T_E and T_W are the temperatures of the east and west piers, measured on the inside about a foot below the pivots;

T'_E and T'_W are similar temperatures measured about 2 feet above the ground;

A, B, C, D are constants.

In order to reduce the rapid changes of temperature it was decided to try the effect of screening the piers from direct air currents. For this purpose felt was used, being a good thermal insulator and the material most readily available at the time.

The Greenwich instrument differs from the Cape instrument in that the circle-reading microscopes are not water-jacketed. The superstructure and the bearings of the Greenwich instrument are open to direct contact with the air. Those exposed metal surfaces were jacketed, as far as possible, with the felt. To shield the piers from direct sunlight, portable screens had been constructed. These were made of sheets of "masonite"—a wood pulp compressed on to a canvas backing—mounted on an aluminium framework. Each screen was in two sections, easily fixed together by means of clips; they protected not only the piers, but also the microscopes, both circles and the pivots. As further protection against rapid changes of temperature, the screens were modified and their inner surfaces were lined with felt. This work was carried out by 1939 November. Data available under these conditions extend only to 1940 September, when observations were discontinued. Observations during this period have shown that the mean daily variation in level has been about halved by this incomplete thermal insulation. It is therefore intended, when the instrument is brought into use again, to provide the piers and the micrometer boxes with a much more efficient thermal insulation. The piers will be protected with slabs of granulated cork, about one inch in thickness, with an external covering. If the diurnal variations in level can be reduced to a negligible amount, the nadir observations will provide a useful check on the variations in collimation.

The level error shows a small seasonal change, with a total range of about $1''{\cdot}5$ during the year. The extreme values occur about the end of March or early April and the end of September or early October, and therefore show a lag, relative to the extremes of air temperature, of six weeks or two months. The ground temperature on the west side of the pavilion is higher during the summer months than that on the east side. It is probable that the small seasonal change in level is caused by a differential expansion between the east and west sides of the foundations which support the instrument.

The direction of the seasonal variation in level corresponds to the west bearing being slightly higher during summer than during winter, relatively to the east bearing. The seasonal change in level is determined from the first observations of level each day, when the instrument is under steady temperature conditions and the temperatures of the two piers are equal.

The mercury trough used for the determinations of level error by means of nadir observations is placed directly on the pier base, below the floor level. A portion of the flooring is removed when the nadir observation is to be made, and a cylindrical screen is placed round the trough to prevent air currents from disturbing the surface of the mercury. When the temperature of the dome is higher than that at the base of the instrument below the floor level, the definition of the wire images is satisfactory. When, however, the temperature of the dome is lower than that at the base of the instrument the nadir observation becomes increasingly difficult the greater the difference of temperature.

With frequent determinations of collimation and level errors, smoothed curves were drawn through the values given by the observations and the adopted collimation and

level errors were read off from these smoothed curves. The drift in clock error in the course of a night, which had previously been observed, was then practically eliminated.

(*g*) *Error of Azimuth.*—As the subsoil at Greenwich is not suitable for stable azimuth marks, such as are used for the control of the azimuth of the transit instrument at the Cape Observatory, the azimuth error of the Greenwich transit instrument must be derived from transits of stars. The main purpose of the instrument being to provide fundamental star places, the most satisfactory method of determining the azimuth is by observations of consecutive transits of the same star above and below pole, termed "double azimuths". Any error that may exist in the adopted R.A. of the star cannot affect the azimuth error, as the place of the star is eliminated. This is the ideal and fundamental method of deriving the azimuth of the instrument, but it assumes that the stability of the azimuth can be relied upon over the period of 12 hours between the upper and lower culminations. The effects of any diurnal variation in azimuth can be minimised by selecting stars whose upper culmination lies near the zenith, so that the azimuth factor is small. With the exception of the close polars, the stars used for double azimuths have been situated between 45° and 78° N. Dec.; in general, the upper culmination is observed during the daytime and the lower culmination at night, either 12 hours earlier or later. The derived azimuth error is then applicable to the night observations. The observations are made with the motor-driven impersonal micrometer, with a speed of drive corresponding to the declination of the star, with the exception of the close polars, *Polaris* and δ Ursæ Minoris. These stars have been observed by travelling the moving wire backwards and forwards at equatorial speed and using the hand tapper to record the instants of its passage over the star image.

Double azimuths are not easily observed at Greenwich where, throughout the year, observations are much interfered with by clouds, mist and smoke. Observations of azimuth stars at lower transit are often impossible because of the smoke over London to the north. There is also a period of some weeks in midsummer when a high Sun and a paucity of bright stars reduces the opportunities of obtaining double azimuths.

When double azimuths have not been observed, the azimuth error must be derived by other methods. The two methods used have been:

(1) A close circumpolar star, either *Polaris*, 51 Cephei, δ or λ Ursæ Minoris, is observed in conjunction with a clock star.
(2) Two circumpolar stars north of declination +70° are observed, one at upper culmination and the other at lower culmination.

Whichever method is used, the transits of the two stars are observed as near together in time as possible.

The azimuth error derived by either of these methods is not independent of the positions of the stars and it is necessary to adopt R.A.'s for the stars. In the preliminary investigations, the R.A.'s of the stars given in the FK3 catalogue were used. The smaller the available azimuth factor, the greater will be the effect of accidental and periodic errors of the adopted R.A.'s on the determination of the azimuth error. These additional methods must be used, in order to supplement the less frequent observations of double azimuths. By comparison between the azimuth errors obtained by the three methods of determination, it is possible to derive corrections to the azimuth errors, in whose determination the star places are not eliminated, in order to reduce them to the fundamental system provided by the double azimuths. This enables all the determinations of azimuth to be used and at the same time to be based on the double azimuth system. The adopted azimuth error for the night is thereby strengthened and the R.A.'s derived from transits become more accurate.

For the nights during the period 1937 September to 1940 September on which azimuth errors were derived by more than one method, the separate determinations of azimuth have been compared in order to find what measure of accordance is given by

the different methods. The comparisons are given in the table below compared under the two headings and grouped as follows:—

(1) Doubles *minus* circumpolars north of +70°,
(2) Close polars *minus* circumpolars north of +70°,

from which can be inferred the difference Doubles *minus* Close polars. The R.A.'s of the two circumpolars, in the circumpolar-star methods, and of the close polar and associated clock star in the close-polar method, have been adopted from the FK3 catalogue.

Azimuth Errors

1937–1940	(1) Doubles – Circumpolars			(2) Close Polars – Circumpolars			(1)–(2)
		Wt.	No. of Nights		Wt.	No. of Nights	Doubles – Close Polars
	″			″			″
Jan., Feb.	−·33	93	14	−·20	93	23	−·13
Mar., Apr.	−·21	97	18	−·09	67	18	−·12
May, June	−·09	155	27	+·05	144	38	−·14
July, Aug.	·00	17	5	+·01	75	22	−·01
Sept., Oct.	+·23	109	22	+·35	94	23	−·12
Nov., Dec.	+·06	115	19	+·08	86	22	−·02
Mean	−·05	586	105	+·04	559	146	−·09

The results indicate that, compared with the azimuths from both doubles and close polars, the azimuths from circumpolar stars north of declination +70° show variations of a periodic nature. There is also a small systematic difference between the azimuths from doubles and from close polars, which may be caused by the different method of observing close polars and stars at a greater difference from the pole. This can be checked in further investigations by observing suitable stars by both methods. Apart from this small systematic error, the accordance between the azimuths from doubles and close polars is very satisfactory; it should be noted that double azimuths are entirely free from errors of the adopted R.A.'s, whilst, in the close-polar method, the effects of such errors on the derived azimuth errors are greatly reduced by the large azimuth factors.

The question arises to what extent the comparisons may be affected by diurnal changes in the azimuth of the instrument. As it is impossible to obtain observational data for azimuth determinations at all hours of the day and night, diurnal changes in azimuth error cannot be controlled as closely as the diurnal changes in collimation and level. From the data available at present, it appears that any diurnal changes in azimuth must be small. Thus, for instance, the double azimuths may be based on observations of a bright close polar, which would be affected by diurnal change in azimuth, or on the observations of a circumpolar whose upper culmination is near the zenith, which would not be appreciably affected by any such change. The azimuths derived in the two cases are in close agreement. Again, in the course of a night's observations, several determinations of azimuth error may be available, extending over several hours. There is no definite evidence of any systematic tendency for the azimuth error to change in a particular direction. From the design of the mounting of the telescope rapid changes in azimuth would not be expected.

The possibility of a systematic drift in azimuth during the day was investigated in another manner. The differences in observed R.A.'s of clock stars and the R.A.'s of the FK3 catalogue were collected according to the U.T. of the observation. The means of such differences for a complete year, for each hour of U.T. covered by the observations,

were formed. Such means are practically independent of any errors in the FK3 places. The means were found to be practically constant up to 18^h U.T., but then showed a slight progressive drift up to 1^h U.T., which could be interpreted as the effect of a drift in azimuth amounting to $0''.2$. But on treating stars transiting within $10°$ of the zenith in the same manner, a similar drift, of equal amount when expressed in time, is shown by north and south zenith stars separately. The drift cannot therefore be produced by diurnal change in azimuth. Though its cause is obscure, it can be concluded that the mean diurnal change in azimuth is satisfactorily small.

Errors of a periodic nature in the right ascension system of the FK3 catalogue would not affect the determinations of azimuth by the double-azimuth method, as errors of star places do not enter into these, nor the determinations by the observation of a close polar and associated clock star, as the R.A.'s of the two stars are nearly the same. But they would affect the determinations of azimuth by the circumpolar method, for the R.A.'s of the two stars—one of which is observed at upper and the other at lower culmination—differ by approximately 12 hours. It is possible therefore that the systematic variation through the year between the azimuths given by the circumpolar observations and those given by double azimuths or close polar observations are caused by periodic errors in the R.A. system of the FK3 catalogue.

The determinations of azimuth by the double-azimuth method are adopted as the fundamental basis of the azimuth. But to discard all transits of azimuth stars for position on nights when no double azimuths are observed would be wasteful. All determinations of azimuth will therefore be reduced to the basis of the double azimuths by the application of the following corrections, which are of a provisional nature:—

(1) To azimuths from a close polar and an associated clock star, a systematic correction of $-0''.10$.

(2) To azimuths from circumpolar stars north of declination $+70°$ corrections of $-0''.30$ in Jan., Feb.; $-0''.20$ in Mar., Apr.; $-0''.10$ in May, June; $0''.00$ in July, Aug.; $+0''.20$ in Sept., Oct.; $+0''.05$ in Nov., Dec.

Future observations will provide data for the revision, if necessary, of these corrections.

The azimuth errors show a small systematic annual variation, with a total range of about $1''.5$, approximately equal to the annual variation in level. The extreme values occur at about the end of April and the end of October.

The seasonal variation in azimuth having been allowed for, the individual azimuth errors for the period 1938·80 to 1940·68 were arranged according to the direction of the wind, under the headings N, NNE, NE, ENE, etc. The wind directions are only approximate and are liable to variations during the course of a few hours. The weighted means by threes were therefore formed and adopted as the values for the mean direction. The results are given in the following table:—

Corrections to be applied to Observed Azimuths depending on Direction of Wind

Direction	Correction	Wt.	Direction	Correction	Wt.
	″			″	
N	+·33	25	S	−·05	23
NNE	+·26	20	SSW	−·27	45
NE	+·19	18	SW	−·28	49
ENE	+·14	19	WSW	−·32	40
E	+·14	15	W	+·03	25
ESE	+·09	20	WNW	+·05	18
SE	+·06	12	NW	+·22	20
SSE	+·03	10	NNW	+·27	22

A pronounced effect depending upon the direction of the wind is clearly apparent. It should be mentioned that as winds from N. to E. tend to be more frequent in the

spring than at other times of the year, the wind effect on azimuth and the annual variation in azimuth are partially correlated. The two effects are not completely separated in the results given above. A more complete separation could have been effected by a second approximation, but this has not seemed worth while for the present discussion, which is of a preliminary nature.

It may be recalled that the observations with the Cookson zenith telescope have clearly shown a close correlation between the direction (but not the strength) of the wind and the position of the zenith. The extreme effects are obtained with N.E. and S.W. winds, the displacement in the zenith between N.E. and S.W. winds being 0″·11. The observations with the Airy transit instrument have also shown a displacement of about the same amount in the zenith point with winds in these directions.

It is probable that the effects of the wind on the azimuths determined with the reversible transit instrument, and on the zenith point in the observations with the Cookson floating telescope and the Airy transit circle, are due to a tilting of the layers of equal density or temperature in the atmosphere.

It may be as well to stress that, if stable azimuth marks are available for the control of the azimuth of a transit instrument, spurious results will be obtained unless the fluctuations in the collimation are accurately controlled.

Housing of the Telescope and Collimators.—The experience gained in the various investigations with the instrument, which have extended over four years, leads to some comments on the general installation, which may be helpful in the consideration of the design of similar equipment elsewhere.

Observations of the Sun, Venus and Mercury, and of clock and azimuth stars during the daytime are essential in fundamental astronomy. It soon became apparent that the copper-covering of the transit pavilion was heated by the direct rays of the Sun sufficiently to impair the definition of the images in day observations, even when the shutters were open to their maximum extent. Further, the concrete surfacing of the area around the pavilion, and the two collimator houses in the line of the meridian, became heated by the direct rays of the Sun and had a further harmful effect on the seeing.

The importance of a close control over the collimation error is of great importance. Any error in the adopted collimation directly affects the observed right ascensions, but has a further indirect effect because it enters into the determination of the level error by nadir observations and of azimuth by observations of azimuth marks. The collimation is usually assumed to be free from rapid changes, and it is customary for collimation to be determined at infrequent intervals, perhaps once daily or even only once or twice a week. The collimation of the new reversible transit instrument has been found to change appreciably in the course of a day; though the cause of these changes has not yet been found, it is improbable that the instrument is unique in this respect.

The observation of collimation, when the collimators are housed separately from the telescope, is often difficult. The best conditions for observation are obtained with a completely overcast sky. With intermittent cloud they are fairly satisfactory. With a clear sky and the Sun shining, conditions are at their worst. The hot air rising from the heated surface of the concrete surround made the images of the collimator wires diffuse. The collimator housings developed an oven effect, the internal temperatures being different from the outside temperature. This caused diffusion and unsteadiness of the wire images, which was more troublesome the greater the difference between inside and outside temperatures. The effects were particularly noticeable at night after a hot sunny day.

The determination of horizontal flexure for use in the declination observations depends upon observations with the collimators. The accuracy of the result depends almost entirely on the stability of the collimating telescope throughout the period of observation. Experience showed that reliable observations could be obtained only when the sky was

overcast and when the temperatures inside and outside the buildings were the same and remained constant. If an observation had been commenced and the clouds broke and the Sun shone before it had been completed, unsteadiness and bad definition resulted and the observation had to be abandoned.

At the end of 1939 louvred wooden screens, painted white, were fitted to the roofs and to the south and west walls of the collimator houses, in order to improve the conditions and to reduce the oven effect. The screens have helped to give substantially better definition than before, but when internal and external temperatures vary greatly, the observations are still difficult.

Experience with the instrument has clearly shown the great advantages that would be gained if the two collimators were housed in the same building as the telescope. Considerations of expense would no doubt be of some importance if this would entail appreciable increase in the size of the telescope pavilion. But it would have been possible without increase in size of the reversible transit circle pavilion to mount the two collimators in the pavilion by making them in the form of bent telescopes, with mirrors placed at a distance of about 18 inches behind the objectives to turn the rays of light through a right angle. There would be sufficient space between the north collimator and the pier for the turntable for the reversing carriage. The cost of separate houses for the collimators would be eliminated; time would be saved in the observation of collimation, because the observer would not have to go outside the pavilion and because of the improvement in definition; and the collimation would be determined with increased accuracy.

The purpose of the concrete surround was to carry rain-water from the pavilion away from the concrete pier on which the instrument is mounted and to keep the surrounding ground dry. These are not unimportant considerations and undoubtedly reduce the seasonal changes in instrumental errors. The small seasonal variations in level and azimuth are evidence of this. On the other hand, the concrete surfacing is detrimental to definition. It is a question for consideration whether this disadvantage does not outweigh the advantage of the stability in level and azimuth. In any similar future installation, it is suggested that the concrete surfacing should be at least one foot below the ground and possibly sloping outwards in the form of a ramp, so that the ground almost up to the pavilion could be covered with turf.

Radiation from the heated copper-covering of the pavilion is troublesome for the day observations. The copper-covering is economical from the point of view of maintenance, as it does not require painting, and is attractive in appearance when the green patina has formed. The development of this patina has been much slower than had been expected, but it is now proceeding satisfactorily. The patina should be beneficial to some extent, but it remains for future observations to show how much is gained by it. If radiation from the covering still proves troublesome, it will be necessary to consider whether the covering should not be painted white and, possibly, the moving shutters covered with light louvred wooden screens, painted white.

AN ANALYSIS OF THE MOTIONS OF CEPHEID VARIABLE STARS, WITH REFERENCE TO GALACTIC ROTATION AND ABSORPTION

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Summary

The radial velocities of the nearer Cepheids have been analysed for solar motion and the Oort double-wave term due to differential rotation. The distances of the objects had been corrected for interstellar absorption, uniform in all directions, but it was found that the Oort constant A was significantly smaller for objects in regions towards the galactic centre than in other directions. This is interpreted as an indication of greater absorption towards the centre than in other longitudes.

It was next assumed that the absorption was uniform in a sector of 240° of galactic longitude from 25° to 265°, and the absorption coefficient was determined by requiring that for objects within 2500 parsecs the Oort constant A shall be independent of distance. This gives an absorption coefficient of 0·52 magnitude per kiloparsec in this sector, and the value of A was found to be 19·1 km./sec. per kiloparsec. If in the sector towards the galactic centre the absorption is also assumed to be uniform, and if the value of A is to be the same here, the absorption coefficient must be 1·36 magnitudes per kiloparsec.

With this simple distribution of absorbing material the distances of 142 Cepheids were calculated. By means of the exact formula for the radial velocities, the variation with galactic radius of the mean angular speed of galactic rotation was found for a range of 5000 parsecs. The galactic centre was estimated to be at a distance of 9030 ± 840 parsecs in the direction of longitude $325°{\cdot}4 \pm 1°{\cdot}7$.

The proper-motions in galactic longitude and latitude of 85 Cepheids were also examined. The great range of distances caused wide variations in the calculated correction to the absolute magnitudes according as the errors in the equations were attributed to linear velocity-dispersion, or to errors of observation in the proper-motions. The weighted mean correction from three such solutions was a decrease in brightness of $0{\cdot}11 \pm 0{\cdot}19$ magnitude, so that in effect the present zero-point of the period-luminosity law is confirmed.

From the proper-motions in galactic longitude a value of 370 ± 43 km./sec. was obtained for the Sun's speed of galactic rotation.

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1. *Introduction.*—The radial velocities and proper-motions of Cepheid variable stars are particularly well suited for the study of galactic rotation for three reasons: the objects are concentrated towards the galactic plane, the peculiar motions are small, and the absolute magnitudes are readily obtained from the period-luminosity law. Moreover, an analysis of these motions yields information not only about galactic rotation, but also about the absorption of light in interstellar space, and about the zero-point of the period-luminosity law, which determines the absolute scale of distance. These three problems are indeed so closely interwoven that it would be difficult to treat any one of them separately.

The method of obtaining statistical parallaxes by using the parallactic component of solar motion, or by combining data from radial velocities and proper-motions, has been in use for many years, and seems to have been sufficiently accurate for all the nearer stars, irrespective of their type. However, if this method is applied to stars as distant as Cepheids, a number of difficulties, both of a theoretical and of a practical nature, are revealed. The paper which follows describes an investigation of the motions of Cepheid variables which started on orthodox lines, but which finally needed some drastic changes before any results of value could be obtained.

The first difficulty is that for distant objects the parallactic component of solar motion ceases to be the dominant factor in proper-motions and radial velocities. Terms in the proper-motions due to differential galactic rotation remain of the same order as the distance increases, but the effect of solar motion varies inversely as the distance. At a thousand parsecs the two factors are comparable, but at greater distances the rotational terms become dominant. Were galactic rotation the only new feature introduced by increasing distances, suitable modifications to the existing method might well be made; Strömberg * has in fact made such corrections. However, the effects of galactic absorption also become important. For example, it may be supposed that the absorption amounts to 0·8 mag. per kpc.; if the absolute magnitude of a group of stars at about 100 parsecs is found, no correction being made for absorption, the calculated absolute magnitude will be too faint by 0·08 magnitude. If the same calculation is made for objects at 1000 parsecs, the error will be ten times as large. Now the uncorrected distances for some of the Cepheids exceed 10 kiloparsecs, and the influence of galactic absorption at this distance will be enormous.

Correction must therefore be made for such absorption, and the amount is determined from the material itself. This is absolutely necessary, for most of the determinations of absorption have been made with material outside the absorbing layer—for example, Hubble's determination from counts of extra-galactic nebulæ.† It is also considered that the investigations on selective absorption, associated with the names of Stebbins, Huffer and Whitford ‡ among others, are of little direct use here, for the relation between selective and total absorption is not reliably known.

In the same way, the other constants entering into the problem are all calculated from the material itself, except where the paucity of the material causes the value of some unknown to be relatively indeterminate. It has long been established that mean motion and velocity-dispersion both vary with spectral type §, so that the practice of carrying over kinematic constants from one investigation to another can only be justified by expediency.

2. *The Data.*—The material for the investigation is taken from papers by A. H. Joy ‖ and R. E. Wilson.¶ Joy's paper tabulates the periods, galactic longitudes and latitudes, apparent magnitudes, photometric distances and radial velocities of 156 Cepheids. There are also the distances corrected for Joy's estimate of galactic absorption, namely 0·85 mag. per kpc.; these have been used provisionally, but it should be pointed out that Joy's figures for objects outside the absorbing layer are incorrect, even if his estimate of the thickness of the layer is adopted.

Of these 156 objects, Joy omits 18 from his analysis for a variety of reasons. This number has been reduced to 14, given with the reasons for their rejection as follows: BB Herc, because of its abnormally high residual radial velocity of over 90 km./sec.; W Virg, because of its high galactic latitude; BH Ophi, BL Herc, CN Lyra, GL Cygn,

* *Ap. J.*, 84, 567, 1936.
† *Ap. J.*, 79, 8, 1934.
‡ *P. Washburn O.*, 15, 217, 1934; also *Ap. J.*, 90, 209, 1939; 91, 20, 1940, and 94, 215, 1941.
§ Strömberg, *Ap. J.*, 61, 379, 1925.
‖ *Ap. J.*, 89, 356, 1938.
¶ *Ap. J.*, 89, 218, 1938.

V343 Cygn, MZ Cygn, SW Taur, 176·1932 Cmaj, AL Virg, RX Libr, V377 Sgtr and V410 Sgtr, which, according to Joy, are outside the absorbing layer. This he assumed to be of total physical thickness 400 parsecs, or of optical thickness, 0·34 mag. However, this is in contradiction to Hubble's estimate of 0·5 mag. for the optical thickness, so it seems more reasonable to leave the physical thickness unknown until the actual absorption within the layer has been found. In any case since these twelve objects are the twelve furthest from the galactic plane, their inclusion might affect the solution adversely, and could at best improve it only by increasing the number of objects used. The four objects now included which had been rejected by Joy were objects at uncertain distances. In all four cases the residual radial velocity is small, so there seems no strong reason to doubt the original estimates.

The proper-motions are taken from a paper by R. E. Wilson *, in which the apparent magnitudes, periods and components of proper-motion are given for 86 stars. Again W Virg is omitted from our analysis, as the high galactic latitude of this star would cause it to be dominant when the motion normal to the galactic plane is considered.

3. *The use of a Scale-Factor.*—Since absorption affects the estimates of relative distance, it is essential to evaluate it first. Any variation in the zero-point of the period-luminosity law only affects the linear scale, and the relative distances remain the same. For instance, if we assume the zero-point of the period-luminosity law, the distance r of a star would be calculated from the absolute magnitude, M (given by the period-luminosity law), and the apparent magnitude, m, by means of the formula,

$$5 \log r = m - M + 5. \qquad (1)$$

If the zero-point is incorrect, the Cepheids may have been assumed to be too faint by an amount, ΔM, and in this case the true distance, r_t, is given by

$$5 \log r_t = m - M + \Delta M + 5. \qquad (2)$$

Hence we have

$$r_t = r(10)^{\Delta M/5}.$$

It follows that the distances calculated from (1) will be correct, except that they will not be measured in true parsecs; instead, until the correction ΔM can be found, we have used an arbitrary unit, written as a (?) *parsec*. The relation between the two units is expressed by the equation,

$$1 \text{ (?) } parsec = (10)^{\Delta M/5} \text{ true parsecs.}$$

If ΔM is zero, that is, if the present zero-point is correct, then the two units are identical.

The correction of distances for interstellar absorption is made in a similar way. If the absorption is k magnitudes per true parsec, the true distance is given by the equation,

$$5 \log r_t = m - kr_t - M + \Delta M + 5;$$

substituting in this for r_t, we obtain the relation,

$$5 \log r = m - \{k(10)^{\Delta M/5}\} r - M + 5. \qquad (3)$$

This corresponds to absorption of amount $k(10)^{\Delta M/5}$ magnitudes per (?) *parsec*. Until the analysis of the proper-motions is completed, the correction ΔM remains unknown, and the unit of distance continues to be the unknown unit, the (?) *parsec*.

4. *The preliminary Analysis of the Radial Velocities, and the Variation of Absorption with Longitude.*—The presence of galactic absorption is revealed when we apply the Oort approximation to the radial velocities.† If r is the distance of an object from the

* *Loc. cit.*

† *B.A.N.*, 3, 275, 1927.

Sun, and l, b are its galactic longitude and latitude, then ρ, its radial velocity, is given in terms of constants u, v, w, and A by the approximate formula,

$$\rho + u\cos(l-l_0)\cos b + v\sin(l-l_0)\cos b + w\sin b = Ar\sin 2(l-l_0)\cos^2 b. \qquad (4)$$

In this, l_0 is the longitude of the galactic centre, provisionally assumed to be 325°. The constants u, v, w are the components of solar motion, and A is the Oort rotational constant. The physical nature of u, v, w needs further explanation; the theory of galactic rotation assumes that for any type of galactic object there is at each point a mean motion which is parallel to the galactic plane and perpendicular to the direction to the galactic centre. This mean velocity, Θ, is taken to be a function of R, the galactic radius, only, for rotational symmetry is assumed, and any variation perpendicular to the galactic plane will necessarily be small, at least in the case of the Cepheids, for they all lie near to the galactic plane. If R_0 is the Sun's distance from the galactic centre, $\Theta(R_0)$, usually written Θ_0, is the mean motion at the Sun's distance for the type of object considered. The quantities u, v, w are the components of the Sun's motion relative to Θ_0, u being towards the galactic centre, v in the direction of rotation, and w perpendicular to the galactic plane. It follows that the components of the Sun's motion relative to a non-rotating frame are u, Θ_0+v, w. From Strömberg's discovery * of the asymmetry of stellar motions we deduce that v certainly changes with spectral type, and so Θ_0 must show similar variation.

In terms of Θ the exact formula for ρ, to which (4) is the approximation, is written,

$$\rho + u\cos\phi\cos b + v\sin\phi\cos b + w\sin b = R_0\left\{\frac{\Theta}{R} - \frac{\Theta_0}{R_0}\right\}\sin\phi\cos b. \qquad (5)$$

In this, ϕ has been written in place of $(l-l_0)$. The right-hand side of (5) has elsewhere † been written in the form, $f(R, R_0)\sin\phi\cos b$, and this definition of f will be used again in the present work. Equation (4) is obtained from (5) by two simplifications; for $\left\{\frac{\Theta}{R} - \frac{\Theta_0}{R_0}\right\}$ is written $\frac{d}{dR}\left(\frac{\Theta}{R}\right)(R-R_0)$, and for $(R-R_0)$ we may substitute $(-r\cos\phi\cos b)$. The first part of the approximation assumes that terms involving higher derivatives of Θ/R can be neglected—an assumption which may not be valid if the curvature of the curve of Θ/R as a function of R is great. The second part involves a systematic error, for it will be seen from a figure that for $|\phi| < 90°$, $r\cos\phi\cos b > (R_0 - R)$, whereas for $|\phi| > 90°$, $-r\cos\phi\cos b < (R-R_0)$. For objects at 2000 (?) *parsecs* the error in $(R-R_0)$ could amount to as much as 10 per cent., and as objects at large distances carry most weight in solving (4) for A, it is essential that a limit be set on the range of r to which (4) is applied. In his preliminary analysis, Joy applied the approximate formula to stars at distances exceeding 5000 parsecs, and the error thus introduced would add to that produced by the then uncorrected distances, causing the constant A to have smaller values with increasing distance. In the present investigation the range was restricted to $r < 2500$ (?) *parsecs*. This is thought to be too large, but in this analysis at least it cannot be much reduced without seriously cutting down the number of objects available.

The distances provisionally used were the corrected distances given by Joy, who had adopted a value of the absorption of amount 0·85 magnitude per 1000 (?) *parsecs*. (Since the zero-point is still unconfirmed all the units are (?) *parsecs*.) Equation (4) was solved for u, v, w, and A for 111 objects, all at distances not exceeding 2000 (?) *parsecs*, and the residual radial velocity of each star was found. This differs from Joy's procedure, since he assumed the solar motion to be the standard value relative to the nearer stars; such a step is scarcely necessary, for it has been emphasized that solar

* *Ap. J.*, 61, 379, 1925.
† *M.N.*, 99, 82, 1938.

motion varies with spectral type, and further, the material is sufficient to determine values of u and v, though the component w is found to be indeterminate.

Now it has been pointed out by many writers * that the absorbing material is anything but uniform. In the present problem there was no hope of establishing any detailed variation, but it seemed advisable to examine the residuals to see if there were any marked variation with galactic longitude. The residuals suggested that for objects towards the galactic centre the value of A was too large, and for those in the opposite direction the value was too small. This was confirmed by dividing the stars into three groups according to longitude, and finding the value of A for each. The values of u, v, w were taken in each case to be those found in the previous analysis of the combined groups. The results of this analysis by groups are given in Table I.

TABLE I

The Apparent Variation of A with Galactic Longitude

Group	I	II	III
Range of longitude	265°–360° 0°–25°	25°–85° 205°–265°	85°–205°
Number of objects	36	35	40
A, in km./sec. per (?) *kpc*	16·4 ±1·8	24·2 ±1·9	22·1 ±1·3

It will be seen that Group I consists of objects lying within 60° of longitude of the direction to the galactic centre, Group III contains the objects in the opposite sector—that is, away from the galactic centre, and Group II consists of material in two sectors, each roughly at right angles to the direction to the galactic centre. The difference between the values of A for Groups II and III is amply covered by the probable errors, but Group I gives a value which is significantly different, and the difference seems too great to be attributed to any breakdown of the Oort approximation. It suggests that the distances of stars towards the centre have been systematically overestimated—that is, that the absorption is greater towards the centre than in other directions. This result is not altogether unexpected, for Alter † and others have already suggested a variation of absorption with longitude. Unfortunately the material from the Cepheids is not sufficient to examine the relation between absorption and longitude in greater detail. It is simply assumed that in the sector of 120° between longitudes 265° and 25° the absorption is uniform, and that outside that sector the absorption is also uniform, but has a different value. The two values must be relatively adjusted so as to give the same value of A from both sectors.

5. *The Determination of the Absorption Coefficient.*—In theory, it is possible to make an absolute determination of the absorption coefficient in the following way. For some selected spectral type we choose a group of near objects and also a group of remote ones; for each group the mean distance is calculated from the radial velocities and proper-motions. Then the mean absolute magnitude, M, and the absorption coefficient, k, can be found from two equations,

$$M + kr_i = m_i + 5 - 5 \log r_i,$$

in which m_i and r_i are the apparent magnitudes and calculated distances of the two groups. In practice this method has always failed for a variety of reasons. For instance it is important that the two values of r shall be as different as possible, and yet within either group the distances must be as nearly equal as possible. But probably the most

* Stebbins, Huffer and Whitford, *loc. cit.*, also Hubble, *Ap. J.*, 79, 44, 1934, and O'Keefe, *Ap. J.*, 94, 353, 1941.

† *M.N.*, 102, 211, 1942, also Zug, *L.O.B.*, 16, 138, 1933.

serious difficulty is that of determining the statistical parallax of a distant group of stars.

It is therefore necessary to use a differential method. This implies that we select some property varying with distance which we would expect the stars to have. Then the amount of absorption assumed is made to vary until the expected property is most clearly revealed. For example, Bottlinger and Schneller * have supposed that the mean distance, without regard to sign, from the galactic plane should be independent of the distance from the Sun. This is clearly a reasonable assumption for stars such as the Cepheids which are concentrated towards that plane, though it might be expected that the mean distance would increase slightly in regions towards the galactic centre. Unfortunately this is a difficult criterion to use, for it would be very susceptible to the thickness of the absorbing layer, and the exclusion of objects believed to be outside the layer might seriously affect the result.

In this analysis, as in Joy's, the absorption is estimated by finding that amount which will secure the greatest uniformity in the values of A at different distances. Two methods were used, and each was applied to the radial velocities of 75 Cepheids within 2500 (?) *parsecs* of the Sun, and lying in longitude between 25° and 265°—that is, the objects in Groups II and III combined.

From equation (3), if the absorption is assumed to be k mag. per (?) *parsec*, the distance of a star of apparent magnitude m is given by the relation,

$$5 \log r = m - kr - M + 5.$$

If, however, the true absorption is $k(1+s)$ in the same units, the distance is not r, but $r(1-ps)$ and we have

$$5 \log r + 5 \log (1-ps) = m - kr(1+s)(1-ps) - M + 5.$$

We may assume that s is small, so that, to a first approximation, we have

$$5ps/\log_{10} e = krs(1-p),$$

so that,

$$1/p = 1 + 5/(kr \log_{10} e). \tag{6}$$

It follows that if k is assumed, the quantity p can be calculated for each object.

With the new value of the distance, equation (4) is written,

$$\begin{aligned} \rho_c &= 2Ar(1-ps) \sin \phi \cos \phi \cos^2 b, \\ &= (A)r \sin 2\phi \cos^2 b - (As)rp \sin 2\phi \cos^2 b. \end{aligned} \tag{7}$$

In this, ρ_c is written for the radial velocity corrected for solar motion.

Method I.—Equation (7) is solved directly for the two unknowns A and As, and the change in the absorption, s is found from the ratio. It is essential that the correction s should be small, otherwise the approximation which gave rise to (6) is no longer valid. The method has been applied to obtain corrections to three assumed values of the absorption, namely 0·50, 0·85 and 1·50 mag. per (?) *kiloparsec*. Only in the first case is the correction small enough to have any significance. The details of the three trials are given in Table II.

Method II.—In the second method the same objects in Groups II and III were divided into five subgroups at different distances. Had the correct value of the absorption been adopted, the value of A for each group would be given by the equation,

$$A = \frac{\Sigma \rho_c r \sin 2\phi \cos^2 b}{\Sigma r^2 \sin^2 2\phi \cos^4 b}. \tag{8}$$

When the absorption coefficient is corrected by a small fraction s, the distances are all

* *Z. Ap.*, 1, 340, 1930.

reduced from r to $r(1-ps)$; this would change the value of A, and the corrected value would be,

$$A_c = \frac{\Sigma \rho_c r(1-ps) \sin 2\phi \cos^2 b}{\Sigma r^2 (1-ps)^2 \sin^2 2\phi \cos^4 b}.$$

As in Method I, it is assumed that s is small, so this can be written in the form,

$$A_c = A + s\frac{dA}{ds}, \tag{9}$$

in which A has the value calculated from (8), and $\frac{dA}{ds}$ is given by

$$\frac{1}{A}\frac{dA}{ds} = \frac{2\Sigma r^2 p \sin^2 2\phi \cos^4 b}{\Sigma r^2 \sin^2 2\phi \cos^4 b} - \frac{\Sigma \rho_c r p \sin 2\phi \cos^2 b}{\Sigma \rho_c r \sin 2\phi \cos^2 b}.$$

The 75 objects in Groups II and III were divided into five subgroups according to distance, and for each subgroup the quantities A, $\frac{dA}{ds}$, and the probable error of A were found. These were then substituted in (9) so as to obtain five equations for the two unknowns, s and A_c, so that in effect we are seeking a value of s which will ensure that all five subgroups give the same value of A. The equations were weighted inversely as the probable errors of A and solved by the usual method of normal equations.

Method II has also been used to correct three assumed values of the absorption coefficient, namely 0·50, 0·85 and 1·50 mag. per (?) *kiloparsec.* These results are also given in Table II.

TABLE II

The Coefficient of Interstellar Absorption. in Magnitudes per (?) kiloparsec.

Assumed k_0	0·50		0·85		1·50	
Method	I	II	I	II	I	II
s	+0·03	−0·37	+0·49	−0·75	+0·66	−1·36
$k=k_0(1+s)$	+0·51	+0·32	+1·27	+0·21	+2·49	−0·54
p.e. of k	0·11	0·14	0·50	0·94	0·35	0·89

The weighted mean of all six values of k was $0{\cdot}56 \pm 0{\cdot}19$ mag. per (?) *kpc.*, but the last three solutions yielded such large values of s that they should be rejected. For the first three, the weighted mean value of k came out to be $0{\cdot}46 \pm 0{\cdot}08$, but it was considered sufficient to adopt the value $0{\cdot}5 \pm 0{\cdot}1$ mag. per (?) *kpc.* for the mean absorption in the sector from 25° to 265° in galactic longitude. The corresponding value of A came out to be $19{\cdot}1 \pm 1{\cdot}2$ km./sec. per (?) *kpc.*, which is to be compared with the value of $20{\cdot}9 \pm 0{\cdot}8$, given by Joy.

From Table II it will also be seen that, though Method I gives the smaller probable error, it does not give consistent corrections. In all three cases the method suggests an increase in the absorption, but the larger the assumed absorption, the larger is the correction. The reader may also notice that if the sign of s were reversed in the three solutions obtained by Method I, the three values of k would all lie between 0·4 and 0·5 mag. per (?) *kpc.* This must be quite accidental, for the work has been carefully checked at this point, and the signs are correct as given.

The absorption in the sector of 120° towards the galactic centre was chosen to give the same value of A, and the value thus obtained is 1·3 mag. per (?) *kpc.* It is not suggested that the absorbing material is really distributed in such a way that in the sector of 120° of galactic longitude from 265° to 25° the absorption is uniformly 1·3, whereas in the 240° from 25° to 265° it is only 0·5 mag. per (?) *kpc.* This is recognized

as an approximation, to allow for the observed result that in the sector towards the galactic centre the radial velocities seem systematically too small, a result which is interpreted as an indication of increased absorption in that area. A uniform increase throughout the area is the simplest correction to make, and the material available for analysis is not sufficiently extensive to establish any detailed variation within the area. The abrupt change in absorption at 25° and 265° in galactic longitude is not as serious as might have been expected, for only ten of the Cepheids have longitudes within ten degrees of the two critical values.

It would be difficult to compare this estimate of the amount of interstellar absorption either with the results obtained by considering colour-excess, or with the results of Hubble *, van de Kamp †, or Oort ‡, which are all based on material outside the layer. Anticipating the results of the investigation on the proper-motions, we may say that this absorption coefficient comes out to be 0·52 mag. per true kiloparsec except towards the centre. Four comparable results are those of Joy, Trumpler §, R. E. Wilson || and Berman ¶, namely 0·85, 0·67, 0·65 and 0·55 mag. per kiloparsec. When it is remembered that in each of these four determinations some of the material used came from the region of greater absorption, the value obtained in this present analysis seems to be in satisfactory agreement. It may also be recalled that Seares ** has pointed out that if there is a *uniform* absorption exceeding 0·7 mag. per kpc., then we are obliged to conclude that the star-density increases in all directions as we go out from the Sun. According to our result, there should be a large increase in density towards the centre, but in other directions the density may well decrease.

6. *The Analysis of the Radial Velocities.*—The radial velocities were provisionally corrected for solar motion, as found in the preliminary analysis of § 4. The distance of each object was corrected for interstellar absorption, as found in the previous section. The position of the galactic centre was provisionally assumed to be at 10 (?) *kpc.*, in the direction of galactic longitude 325°; R, the distance of each star from the galactic axis, was calculated from the formula,

$$R^2 = R_0^2 + r^2 \cos^2 b - 2R_0 r \cos b \cos \phi.$$

The objects were arranged in order of galactic radius, and conveniently grouped.

Equation (5) can be written in the form,

$$\rho_c = f(R, R_0) \sin \phi \cos b. \qquad (10)$$

From this formula the value of f was found for each group, corresponding to a mean value of R for the group. In this way the relation between f and R can be represented as a curve, as in Fig. 1, in which the continuous line is the final graph of the function. The analysis was not yet complete, for corrections still had to be made to the assumed values of u, v, w, R_0 and l_0.

The correction to the position of the galactic centre was obtained by supposing that the preliminary curve of f is correct, but that the distances had all been measured from the wrong centre. If the true centre were at a distance $(R_0 + \delta R_0)$ in a direction of longitude $(l_0 + \delta l_0)$, then equation (10) takes the form,

$$\rho_c = f(R, R_0) \sin \phi \cos b + \delta R_0 \frac{\partial}{\partial R_0}\{f(R, R_0)\} \sin \phi \cos b + \delta l_0 \frac{\partial}{\partial l_0}\{f(R, R_0) \sin \phi\} \cos b.$$

In this, the value of f and its derivative with respect to R are read off from the preliminary curve.

Similarly the radial velocities were only corrected approximately for solar motion,

* *Loc. cit.*
§ *L.O.B.*, 14, 154, 1930.
** *Ap. J.*, 74, 91, 1931.
† *A.J.*, 42, 161, 1933.
|| *Ap. J.*, 92, 170, 1940.
‡ *B.A.N.*, 8, 233, 1938.
¶ *L.O.B.*, 18, 69, 1937.

so that terms for the unknown corrections to the adopted value must also now be added. In the same way it is usual to reduce all the radial velocities by a constant, the K-term, chosen from the material itself so as to minimize the velocity-dispersion. Thus there were six unknowns, namely the K-term, and the corrections u', v' w', δR_0 and δl_0, and we have for the corresponding equation of condition,

$$\rho_c + u' \cos\phi \cos b + v' \sin\phi \cos b + w' \sin b - K = f \sin\phi \cos b + \delta R_0 \frac{\partial f}{\partial R_0} \sin\phi \cos b + \delta l_0 \frac{\partial}{\partial l_0}(f \sin\phi) \cos b. \quad (11)$$

This was solved by the method of normal equations, and the values of the six unknowns are given in Table III.

It has been pointed out by Milne * that if a uniform expansion of the galaxy were assumed, then a K-term which was directly proportional to the distance from the Sun would arise in each radial velocity. Equation (11) was therefore also solved in a form modified by replacing K by kr. Again the results appear in Table III.

TABLE III

Constants evaluated in the Analysis of the Radial Velocities

	Constant K-term	K replaced by kr
u } components of	+ 6·1 ± 1·3	+ 6·9 ± 1·4
v } solar motion	+13·0 ± 1·8	+13·3 ± 1·8
w } in km./sec.	+20·4 ± 8·5	+18·8 ± 8·1
l_0	325°·4 ± 1°·7	325°·6 ± 1°·7
R_0 in (?) *parsecs*	9478 ± 223	9558 ± 228
K-term	$K = -1·87 \pm 1·06$ (km./sec.)	$k = -1·3 \pm 0·5$ (km./sec./kpc).
σ, km./sec.	13·6	13·5

The two columns of Table III give the values of various constants determined either with a constant or with a variable K-term. There are no significant differences between the two sets, and the probable errors in the two forms of K-term are sufficiently large to suggest that neither form is really necessary. The constants u, v, w are the components of the Sun's velocity relative to the mean rotation of the Cepheids at the Sun's distance. It was to be expected that w, the component of velocity normal to the galactic plane, would be indeterminate, for the Cepheids are closely concentrated on either side of that plane, and motion normal to it appears in the proper-motions much more than in the radial velocities. The component of velocity parallel to the galactic plane is 14·0 km./sec., towards galactic longitude 30°, to be compared with the standard solar motion, assumed by Joy, which has a component of 18·6 km./sec. in the galactic plane, towards longitude 22°. In his analysis of the radial velocities Wilson † obtained for this quantity a velocity of 27·2 km./sec., towards longitude 40°, but this is partly explained by his inclusion of all the objects, irrespective of distance, in an equation of the type (4), thus assuming the Oort approximation to be valid for all distances. His failure to correct for absorption in any way before solving the equation is also likely to affect the final result adversely.

The velocity-dispersion, σ, is given in the last row of Table III, and agrees almost exactly with Joy's value of 10·8 km./sec. for the average residual velocity without regard to sign.

The twelve objects which had been excluded as being outside the absorbing layer can now be reconsidered. It has been established that, except towards the galactic centre, the absorption in or near the galactic plane is 0·5 mag. per (?) *kpc*. From counts

* *P.R.S.*, A, 160, 20, 1937. † *Ap. J.*, 89, 226, 1938.

of nebulæ Hubble * has suggested that the total optical thickness of the layer is 0·5 mag., so we suppose a physical thickness of 1 (?) *kpc.* In the sector towards the galactic centre the absorption increases, but the physical thickness is unlikely to be less, and again it is taken to be 1 (?) *kpc.*

With these assumptions about the thickness of the supposed uniform absorbing layer, it is found that seven of the twelve stars are inside the layer, and five, namely

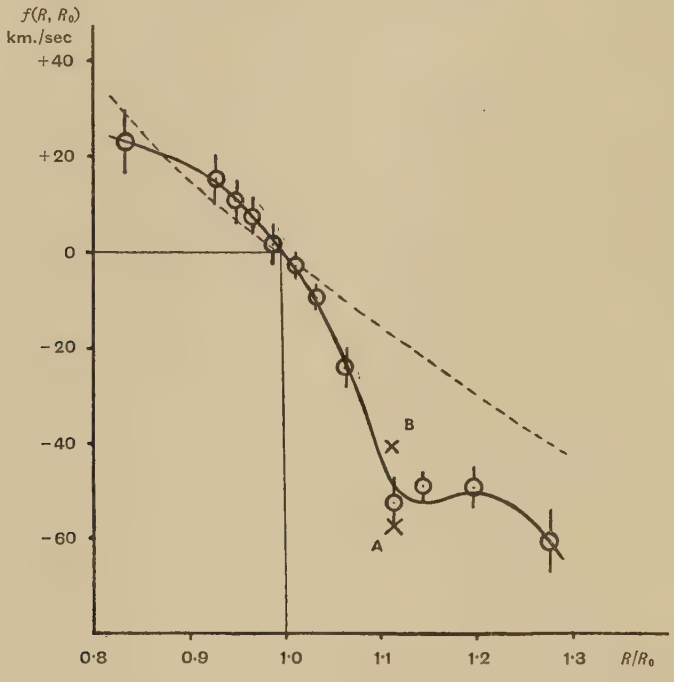

FIG. 1.—*The relation between $f(R, R_0)$ and R.*

MZ Cygn, AL Virg, RX Libr, V377 Sgtr and V410 Sgtr, are outside. The distances of the seven were calculated from (3), taking the appropriate value of k. For the objects outside the layer we have that the length of light-path within the layer is $1/(2 \sin b)$ (?) *kpc.*, where b is the galactic latitude. Thus the distances are given by the relation,

$$5 \log r = m - \tfrac{1}{2}k \operatorname{cosec} b - M + 5,$$

in which k is the appropriate absorption in magnitudes per (?) *kpc.*, either 0·5 or 1·3.

For each star the distance R from the galactic axis was found, and the value of $f(R)$ read off from the curve. Thus the residual radial velocities after correction for solar motion and galactic rotation were found. The root-mean-square residual is 31 km./sec. for the twelve, as compared with 13·6 for the 142 Cepheids originally used. It is rather remarkable that for six of the twelve the dispersion is only 13·9 km./sec., whereas for the others it is over 41 km./sec. It is also noteworthy that five of these six large residuals would be lowered by a decrease in the amount of absorption assumed.

The corrections which had been calculated to add to the provisional values were all made, with the exception of the change in direction of the galactic centre, which only amounted to 0°·4, and seemed too small to affect the result in any way. With the new values the curve of Fig. 1 was recalculated as the final relation between $f(R)$ and R. The straight line drawn through each of the twelve points gives the probable error of the corresponding ordinate.

* *Ap. J.*, 79, 51, 1934.

The curve has been drawn so as to fit as far as possible the peculiarly large negative value of f at $R/R_0 = 1{\cdot}11$. This was afterwards found to be due to a "group motion" of six stars of the thirteen used in the determination of that point. These six all lie between longitudes 196° and 210°, and their distances range from 1750 to 2060 (?) *parsecs*; treated as a separate group, they determine a value of f indicated by the point A. The remaining seven objects, at longitudes varying from 90° to 160°, and ranging in distance from 930 to 2260 (?) *parsecs*, determine the point B, which would lie on a much smoother curve through the other eleven points. This peculiarity may be quite accidental, but otherwise the explanation seems to be that in the small sector, from 196° to 210° the absorption is much less than the value 0·5 which has been used.

The broken curve which also appears on Fig. 1 is the corresponding curve, previously * obtained, for the planetary nebulæ. There is no reason why the two curves should agree, for it cannot be expected that the mean motions of the two systems will be the same. Yet the comparison is of interest, especially if attention be drawn to the differences in the two analyses. The methods were the same, but there is the outstanding difference that the distances of the Cepheids have been corrected for interstellar absorption, whereas the distances of the planetaries were left uncorrected. Had such a correction been made, the broken curve would have been much steeper, for f would remain about the same for any object, but the difference, $R - R_0$, would be reduced. Clearly this might reduce the dissimilarity for $R > R_0$, but it would lead to considerable differences for $R < R_0$, where there is already reasonable agreement. It therefore appears that the two curves are essentially different, and that no adjustment of the absorption would bring them into agreement, provided that the absorption were taken to apply to both kinds of star.

7. *The Analysis of the Proper-Motions in Galactic Longitude.*—It will already have been realized that the scale-factor of section 3 was not involved in any way in the analysis of the radial velocities, and indeed it is impossible to determine the factor from the radial velocities only. Similarly it may be shown that the proper-motions only do not give the required factor, but the combination of radial velocities and proper-motions is necessary. It is sufficient to combine the proper-motions either in longitude or in latitude with the preceding work, and as the analysis of the longitudinal motions was the more promising, it was carried out first.

The method is simply to equate the theoretical linear velocity, transverse to the line of sight, to the product of the corresponding proper-motion and the distance. For such motion parallel to the galactic plane the exact theoretical mean velocity, corresponding to (5), is the quantity,

$$u \sin \phi - v \cos \phi + f(R)\{\cos \phi - r \cos b/R_0\} - \frac{\Theta_0}{R_0} r \cos b. \qquad (12)$$

All the symbols have the same meaning as in the analysis of the radial velocities. If μ_l is the proper-motion in galactic longitude, corrected for precession according to Newcomb's results †, the true angular velocity is

$$\mu_l \cos b - \Delta p\{0{\cdot}492 \cos b + 0{\cdot}870 \sin (l - 153°{\cdot}2) \sin b\} - \Delta\mu_\alpha\{0{\cdot}470 \cos b - 0{\cdot}883 \sin l \sin b\}. \qquad (13)$$

In this, Δp and $\Delta\mu_\alpha$ are the corrections to Newcomb's constants of precession. To convert this to a linear transverse velocity it is necessary to multiply by r, the distance of the object. If the expression (13) is measured in seconds of arc per annum, the corresponding linear velocity would be obtained in km./sec. by multiplying by $4{\cdot}74\, r_t$, where r_t is the distance in true parsecs. As all the distances used in the analysis so far have been measured in the artificial unit, the (?) *parsec*, the multiplying factor is taken

* *M.N.*, 99, 83, 1938.

† Oort, *B.A.N.*, 8, 149, 1937.

to be λr, where r is the distance in (?) *parsecs*, and λ is an unknown constant, namely $4{\cdot}74(10)^{\Delta M/5}$. Hence the equation of condition takes the form,

$$u \sin \phi - v \cos \phi + f(R)\left\{\cos \phi - \frac{r}{R_0} \cos b\right\} - \frac{\Theta_0}{R_0} r \cos b$$
$$- \lambda r \mu_l \cos b + \lambda r \Delta p\{0{\cdot}492 \cos b + 0{\cdot}870 \sin (l - 153^\circ{\cdot}2) \sin b\}$$
$$+ \lambda r \Delta \mu_\alpha\{0{\cdot}470 \cos b - 0{\cdot}883 \sin l \sin b\} = 0.$$

This can be written more simply in the form,

$$V + \Omega r \cos b + \lambda P r \cos \phi \sin b + \lambda Q r \sin \phi \sin b - \lambda r \mu_l \cos b = 0. \qquad (14)$$

In this, the new symbols are defined as follows:—

$$V = u \sin \phi - v \cos \phi + f(R)\{\cos \phi - r \cos b/R_0\},$$
$$\Omega = -\Theta_0/R_0 + \lambda \Delta p 0{\cdot}492 + \lambda \Delta \mu_\alpha 0{\cdot}470,$$
$$P = \Delta p 0{\cdot}870 \sin 171^\circ{\cdot}8 - \Delta \mu_\alpha 0{\cdot}883 \sin 325^\circ,$$
$$Q = \Delta p 0{\cdot}870 \cos 171^\circ{\cdot}8 - \Delta \mu_\alpha 0{\cdot}883 \cos 325^\circ.$$

In theory, it is possible to solve (14) for the five unknown constants, u/λ, v/λ, Ω/λ, P, Q, and the unknown function, $f(R)$, but this would be useless for our purpose, since λ would not be determined. Instead, the values of u, v and $f(R)$, in known linear measure, are adopted from the analysis of the radial velocities, so that the quantity V in (14) can be calculated for each star. This is in marked contrast to the procedure of Wilson.* As a preliminary step, he attempts to solve (14) for the terms in V and Ω due to galactic rotation, and also for the components of solar motion, u and v. It is scarcely surprising that the Oort constants of galactic rotation, A and B, are found to be almost indeterminate, on account of the unreliable proper-motions. Similarly the components of solar motion were found to be inconsistent with the values obtained from the radial velocities, but this may be due to the fact that Wilson appears to have divided equation (14) by r, with the exception of the two terms involving u and v. (This may also be merely a slip in writing the equation.) He therefore decides to omit galactic rotation completely, and to adopt for the apex of solar motion the value obtained by Wilson and Raymond † in an entirely different investigation. Thus, in effect, he determines V by omitting half of it, and making a reasonable guess at the rest.

The solution of (14) raises a theoretical difficulty at this stage, for the equation is subject to two sources of error. There is a dispersion in linear velocities, so that (12) is a mean velocity only, with a random distribution of peculiar velocities about it. Secondly, the observed motion, μ_l, is subject to errors of observation, errors, moreover, which are quite comparable with the observed value of μ_l, which is usually of the order of $0''{\cdot}01$ per annum. Thus the errors in (14) arise in two distinct terms, and so lead to further complications. It should be noticed that the same problem did not arise in the case of the radial velocities. In (10), for instance, ρ is also subject to an error of observation, but here it is in linear measure, and is readily combined with the natural dispersion about the mean velocity.

Although it might be possible to solve a statistical equation such as (14), in which errors are contained in two distinct terms, it would be hopeless to attempt such a solution in this case, where the equation contains four unknowns and the data is restricted to 85 Cepheids, for which the proper-motions and their probable errors are of the same order of magnitude. Instead, two solutions have been made, in one of which the errors are assumed to be entirely in the measured proper-motions, and in the second of which the errors are taken to be in the linear velocity-dispersion only.

* *Ap. J.*, 89, 227, 1938.
† R.A. $=270^\circ$, dec. $=+36^\circ$ (*A.J.*, 47, 57, 1938).

The deliberate neglect of one source of error is to some extent counterbalanced by appropriate weighting. For instance, assuming the errors to be entirely in the proper-motions, we should solve (14) in the form,

$$r\mu_l \cos b = \left(\frac{1}{\lambda}\right)V + \left(\frac{\Omega}{\lambda}\right)r \cos b + Pr \cos \phi \sin b + Qr \sin \phi \sin b. \qquad (15)$$

The linear dispersion affects the term V on the right-hand side of this equation theoretically to the same extent for each star. This will have the effect of increasing ΣV^2 artificially, and so will give too large a value of λ. The equation could also be divided by r, and written,

$$\mu_l \cos b = \left(\frac{1}{\lambda}\right)\frac{V}{r} + \left(\frac{\Omega}{\lambda}\right)\cos b + P \cos \phi \sin b + Q \sin \phi \sin b. \qquad (16)$$

In this form the errors in V/r will be particularly large for near objects; the balance of the equation is upset, and the solution obtained would fit the data for near stars at the expense of more distant ones. The advantage of (15) over (16) can be put in another way. If the errors are attributed to both sources, it is seen that for distant objects errors in proper-motion are more serious than linear velocity-dispersion. Similarly for near stars the opposite is true. It follows that if we decide to attribute all the errors to proper-motions, we must give greater weight to distant objects and less to nearer ones; that is, the equations must be weighted in the ratio of the distances, as in (15). For similar reasons the equation which assumes the errors to be due to velocity-dispersion only is in angular measure, and takes the form,

$$\frac{V}{r} = \lambda\mu_l \cos b - \Omega \cos b - \lambda P \cos \phi \sin b - \lambda Q \sin \phi \sin b. \qquad (17)$$

The neglected errors of observation in μ_l tend to make $\Sigma\mu_l^2 \cos^2 b$ too large, and consequently λ too small.

The real cause of these complications in assigning errors and weights may be recog nized as arising from the great range of distances, the greatest of which is more than thirty times the least. If the objects were all between 200 and 300 parsecs away, weighting as the distance would make a negligible difference, and we could properly attribute the errors to velocity-dispersion. Or, if they were at distances ranging from 2000 to 3000 parsecs, again weighting would be unimportant, and the errors would largely be in the measured proper-motions. The actual situation is that the distances extend from less than 100 to over 2000 (?) *parsecs*—a range which gives rise to the maximum inconvenience.*

Two further factors affect the weighting. In his paper Wilson points out that the determinations of proper-motion are not all equally reliable, and to each component of proper-motion he assigns a probable error. The equations, whether in the form (15) or (17), are therefore weighted inversely as these probable errors. It was also found that, despite the above method of weighting, the solution could be completely dominated by a few stars only. In the case of (15), these usually had large proper-motions, whereas for (17) a few near objects were dominant. It was therefore considered advisable to give only one quarter of their previous weight to stars for which $\mu_l > 0''.02$ per annum, or for which $r < 250$ (?) *parsecs*. In all, sixteen stars are affected by this.

Equations (15) and (17) were solved separately for Ω and λ, but in both cases it was found necessary to assume values for P and Q, the constants which involve the precessional corrections. The values adopted were those given by Oort †, namely

* Wilson appears to overlook this great range of r (which would actually be even greater in his case, for at this stage he had not corrected for absorption), and uses a mean parallax throughout.

† *B.A.N.*, 8, 149, 1937.

$\Delta p = +0''\cdot0089$ per annum, $\Delta\mu_a = -0''\cdot0088$ per annum, the probable errors being $0''\cdot0010$ in either case. These seemed more reliable than the values given by Wilson and Raymond *, which, though they have a smaller probable error, appear to have been calculated from an equation which did not contain the double-wave term arising from the differential rotation of the galaxy.

The values of λ and Θ_0 obtained from equations (15) and (17) are given in Table IV; as had been expected, (15) gives too large, and (17) too small a value of λ. The large probable error in the first solution is in part due to the same cause. Equation (15) is really solved for $1/\lambda$ and its probable error; the ratio of the probable error of λ to that of its reciprocal is λ^2, so the inflated value of λ has increased the probable error.

8. *The Analysis of the Proper-Motions in Galactic Latitude.*—The same general method was used in analyzing the proper-motions in galactic latitude. The theoretical linear velocity, transverse to the line of sight and normal to the galactic plane, is

$$f(R)\sin\phi\sin b - u\cos\phi\sin b - v\sin\phi\sin b - w\cos b.$$

In this, the constants u and v and the function $f(R)$ are known from the analysis of the radial velocities, but w, the component of solar motion towards the galactic pole, is unknown to all intents and purposes, since the value of $+20\cdot4 \pm 8\cdot5$ km./sec., given in Table III, is really meaningless. The corresponding angular velocity is

$$\mu_b - 0\cdot870\Delta p\cos(l - 153^\circ\cdot2) + 0\cdot883\Delta\mu_a\cos l.$$

This can also be written in the form,

$$\mu_b - (0\cdot870\Delta p\cos 171^\circ\cdot8 - 0\cdot883\Delta\mu_a\cos 325^\circ)\cos\phi + (0\cdot870\Delta p\sin 171^\circ\cdot8 - 0\cdot883\Delta\mu_a\sin 325^\circ)\sin\phi.$$

Thus the equation of condition can be expressed,

$$V' - w\cos b - \lambda r\mu_b + \lambda Qr\cos\phi - \lambda Pr\sin\phi = 0. \qquad (18)$$

Here, P and Q are the constants involving the precessional corrections, first used in (14), and V' is the known linear speed, given by

$$V' = \{f(R)\sin\phi - u\cos\phi - v\sin\phi\}\sin b.$$

The position of the factor $\sin b$ in (18) makes the determination of λ from proper-motions in latitude unreliable, for $\sin b$ is small for almost all the Cepheids—b only exceeds 15° for 4 of the 85 objects used. In (14) low latitudes reduced only the coefficients of P and Q, so that the results obtained were relatively independent of the precessional corrections. On the other hand, the effect on (18) of these small values of b is to make V' small; unfortunately V' is the only quantity in the equation which is known in linear measure, and is thus the critical term in the determination of λ.

The equation of condition was solved in two forms, as in the case of the proper-motions in galactic longitude, the two equations being

$$r\mu_b = \left(\frac{1}{\lambda}\right)V' - \left(\frac{w}{\lambda}\right)\cos b + Qr\cos\phi - Pr\sin\phi, \qquad (19)$$

and

$$\frac{V'}{r} = \lambda\mu_b + w\frac{\cos b}{r} - \lambda Q\cos\phi + \lambda P\sin\phi. \qquad (20)$$

The same system of weighting was also adopted, namely that the weights were inversely proportional to the probable errors of μ_b, except that objects for which $\mu_b > 0''\cdot02$ per annum, or for which $r < 250$ (?) *parsecs*, were given only one quarter of this weight.

* *A.J.*, 47, 52, 1938.

It also seemed advisable to adopt Oort's values of the precessional corrections, so that the constants P and Q were again assumed.

Despite these precautions the solution of (20) was valueless, for λ, the scale-factor, came out to be negative. The solution of (19) is given in Table IV.

TABLE IV

The Determination of λ from the Proper-Motions

Material and Method	λ	Θ_0, km./sec.	w, km./sec.
Galactic longitude, (15), errors in proper-motions	$14{\cdot}4 \pm 5{\cdot}5$	377 ± 194	...
Galactic longitude, (17), errors in velocity-dispersion	$4{\cdot}23 \pm 0{\cdot}39$	356 ± 44	...
Galactic latitude, (19), errors in proper-motions	$3{\cdot}09 \pm 2{\cdot}77$	...	3·84
Galactic latitude, (19), errors in proper-motions	5·39	...	6·85 (assumed)
Galactic latitude, (20), errors in velocity-dispersion	1·49	...	6·85 (assumed)

In Table IV are listed the solutions of the various equations for λ, the numbers of which are given, together with the kind of data and the method used, in the first column. The values of λ obtained from the proper-motions in galactic latitude are very susceptible to the value of w the component of solar motion towards the galactic pole. If w is assumed to be the proper component of the solar motion relative to the near stars, namely 20 km./sec. towards the apex, $\alpha = 271°$, $\delta = 28°$, then its value is 6·85 km./sec. The two corresponding values of λ are given in the last two rows of Table IV.

9. *The Final Values of the Constants.*—The final value of λ was calculated from the first three entries in Table IV. Since λ is a scale-factor, it seemed advisable to take, not the arithmetic mean, but the logarithmic mean. This is equivalent to calculating ΔM from the formula, $\lambda = 4{\cdot}74(10)^{\Delta M/5}$, and taking the arithmetic mean of the three values. The weighted logarithmic mean value of λ came out to be $4{\cdot}51 \pm 0{\cdot}40$, and the corresponding value of ΔM is $-0{\cdot}11 \pm 0{\cdot}19$ magnitude—a negligibly small decrease in absolute magnitude. This confirms the existing scale of absolute magnitudes, and the zero-point of the period-luminosity law, as determined by Shapley * at a time when the extra complications of galactic rotation and absorption had not yet been foreseen. It must be stressed that the value of λ obtained from a single equation in the proper-motions is markedly affected by the weighting system adopted, but the mean value from a pair of equations such as (15) and (17) is rather less sensitive.

Wilson's analysis of the proper-motions leads to the result, in our notation, $\Delta M = +0{\cdot}38 \pm 0{\cdot}22$—that is, the absolute magnitudes should be made brighter by this amount. This result is qualified by a second determination †, which is unfortunately quite useless. In it, he assumes the Oort approximation for the radial velocities, and adopts for A the value of 20·9 km./sec. per kiloparsec which Joy obtained by *assuming* the accuracy of Shapley's zero-point, and in which he had corrected for absorption. This is combined with the mean value of (rA), as obtained from the radial velocities, to give $\bar{r}$, the mean distance, and is finally converted to a mean absolute magnitude, by using apparent magnitudes also corrected for absorption. This second determination, which logically should give a zero change, actually suggests an absolute magnitude fainter by 0·37 mag. On grounds of logic alone this correction must be ruled out.

The weighted mean value of Θ_0, the mean linear speed of rotation at the Sun's distance, came out to be 357 ± 43 km./sec. This is much larger than the usual estimate, for it gives the Sun's rotational speed $(\Theta_0 + v)$, as 370 km./sec., in place of the usual values of 270 or 300 km./sec. However, there have been previous estimates of this order of magnitude and greater.‡

* *Ap. J.*, 48, 89, 1918.

† *Ap. J.*, 89, 238, 1938.

‡ Strömberg, *Mt. W. Cont.*, 13, 195, 1925; also Lindblad, *Ups. Med.*, 4, p. 6, 1926; and Camm, *M.N.*, 100, 48, 1939.

With the quantities λ and Θ_0 known, we can calculate the dispersions, either in linear or angular measure. These are really the square roots of the weighted means of the squares of the residuals. In linear measure the dispersion is 27 km./sec., for each analysis, whether in longitude or latitude, and in angular measure it is $0''{\cdot}007$ per annum. In view of the linear dispersion of 13·6 km./sec. obtained for the radial velocities, this suggests, if suggestion were needed, that the proper-motions are subject to large errors.

Now the scale of distance has been determined we are able to dispense with the artificial unit, the (?) *parsec*, which is equal to 0·952 true parsecs. The distance to the galactic centre is now estimated at 9026 ± 836 parsecs, and the absorption in the two sectors towards and away from the centre is found to be 1·36 and 0·52 magnitudes per kiloparsec respectively.

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A PHOTOGRAPHIC SURVEY OF GALACTIC CLUSTERS: V. NGC 189, I 1590, 358, 366, 381, 433, 436, 457, 609, 637, I 166, 743

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Summary

Among the twelve star groups dealt with in this paper two (NGC 358, I 1590) are considered not to be galactic clusters and two others (NGC 609, I 166) contain only very faint stars. The remainder are dealt with as in former papers of this series, giving for each cluster the co-ordinates, magnitudes and number of stars, the cluster distance and its diameter.

NGC 366 and 637 probably contain some giant stars, and NGC 189, 366, 436, 457 appear to be obscured.

The presence of double or multiple stars in clusters NGC 366, 743 and especially 457 (which contains five systems) suggests the need for further spectral investigations of the relations between clusters and multiple stars.

In the regions or neighbourhoods of NGC 366 and 457 three very red stars with colour indices $\geqq 2^m.0$ were found.

An unusual difference between the numbers of cluster stars in NGC 457, as determined by two different methods, is explained.

The method of this investigation has been described in four earlier papers*, referred to later as A I to A IV.

The present paper contains all star condensations within the inner Cassiopeia region, called clusters in NGC and the Index Catalogues, and not dealt with in the earlier papers. They are distributed between galactic longitudes 89° and 99°, and galactic latitudes $-4°$ and $+3°$. Two of the twelve listed objects were not considered to be clusters, for reasons similar to those given in A IV: NGC 358, no trace of which can be found on any available photograph; and I 1590, coinciding in position with the nebula NGC 281. This nebula, however, contains a multiple star, HD 5005 (ADS 719), composed of 5 stars which cannot be separated on Sidmouth plates.† Among the remaining objects the two following were not suitable for the usual treatment:—

NGC 609 (R.A. $= 1^h\ 29^m.6$, Dec. $= +64°\ 2'$ (1900·0)) appears on Sidmouth plates as a conglomeration of very faint stars; a few of the brightest, of about 14^m, are discernible in a diffuse grey patch of about $2'.5$ diameter representing the cluster. It is $2'.5$ np ADS 1249. (The R.A. of the cluster given by the catalogues is incorrect.) Collinder lists 10 cluster stars and a diameter of $2'.5$; Trumpler describes the cluster as detached and concentrated, diameter $3'$, distance 3430 parsecs.

I 166 (R.A. $= 1^h\ 45^m.6$, Dec. $= +61°\ 20'$ (1900·0)) appears only as a vague indication of clustering of some extremely faint stars on the Sidmouth plates. It was observed only once by Denning (private communication to Dreyer) and is described as small (nebulous?) cluster.

For the magnitude determinations of stars in NGC 189 and 637 the plates of A III

* *M.N.*, 100, 387, 1940; 101, 89, 298, 1941; 103, 10, 1943.

† See also Hubble, *Ap. J.*, 56, 184, 1922.

and A I, respectively, were used. The photographs of NGC 743 were taken on a steady clear night; order of exposures: cluster, NPS. The magnitudes of NPS show a mean deviation of $\pm 0^m.11$ on the photographic scale and of $\pm 0^m.05$ on the photovisual scale; the mean error of the colour index is therefore $\pm 0^m.12$. The photographs of the remaining group of 5 clusters were taken on a breezy night with very thin haze; order of exposures: NPS, clusters. The magnitudes of NPS show a mean deviation of $\pm 0^m.10$ on the photographic and of $\pm 0^m.12$ on the photovisual scale; the mean error of the colour index is therefore $\pm 0^m.16$.

Among the results of this paper there is one suggesting further inquiry; it concerns the number of cluster stars. With NGC 457, the number determined by counting is considerably larger than that determined by colour index. A table giving the equivalent differences for other clusters shows an average surplus of 3 stars for the number determined by counting. This small average difference is explained by the method of using colour indices in determining star numbers. The limits of the $(m-M)$ range, including all assumed cluster stars, are directly related to the mean errors of colour index and spectral type, as shown in A II, and so is the star number, determined by this method. The range, and hence the star number, thus depend on a statistical behaviour, so that a few true cluster stars, having larger deviations either in colour index or absolute magnitude than those determining the limits of the range (the mean deviations), are not included within the range. They appear, however, within the cluster region, and are therefore included in the number counted. The large difference found in NGC 457 is dealt with later.

NGC 189

R.A. $= 0^h\ 33^m.9$, Dec. $= +60°\ 31'$ (1900·0)

The cluster lies within a comparatively rich region bordering the heavily obscured sac within which NGC 225 (A III) is situated. The background density is, however, about half that of the other clusters in the neighbourhood so that some absorption might justifiably be assumed. The cluster is No. 462 in Collinder's catalogue.

Direct counting gave 25 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 2060 stars per square degree and 19 cluster stars within a cluster region of $3'.7$ diameter.

Table I

Co-ordinates, Magnitudes and Distance Moduli of 19 Stars (NGC 189)

Zero point = HD 3672, unit = $1'.0$, X = west, Y = north
Co-ordinates of the assumed cluster centre $X = +0.5$, $Y = +8.2$

Star	X	Y	m_{pg}	m_{pv}	$m-M$	Star	X	Y	m_{pg}	m_{pv}	$m-M$
10	+0·5	+8·1	11·02	10·17	4·7	11	+0·3	+9·7	13·39	13·02	9·9
8	+0·7	+6·8	12·07	11·41	6·7	15	−0·3	+9·2	12·61	12·36	10·1
19	−0·7	+8·0	11·80	11·32	7·6	23	−1·2	+8·4	14·21	13·84	10·7
7	+0·9	+9·7	14·60	13·82	8·6	20	−0·7	+7·9	13·21	13·02	11·1
21	−0·8	+7·0	13·90	13·31	9·0	17	−0·6	+9·6	14·08	13·78	11·2
2	+2·2	+7·6	13·59	13·09	9·3	13	+0·1	+7·2	12·71	12·63	11·4
3	+2·0	+8·3	13·21	12·79	9·4	24	−1·3	+7·7	13·49	13·33	11·6
25	−1·3	+7·9	14·42	13·83	9·5	12	+0·3	+7·8	13·05	13·04	12·3
4	+1·6	+8·8	13·70	13·27	9·9	16	−0·4	+7·6	13·94	13·85	12·6
5	+1·1	+7·5	12·75	12·45	9·9						

Table I gives the distribution of $m-M$, and the mean distance modulus is 10·114. The range of the distance moduli (3·0) is more than should be expected for a star group

with an average colour index of $+0^m.31$ and a mean error of CI of $\pm 0^m.10$. Stars of this CI should fill a range of about 2·0, and the range of 3·0 corresponds to a star group of at least $+0^m.20$ colour excess. This irregularity in the range supports our assumption of absorption, derived from the difference in the background densities. A colour excess of $0^m.20$ puts the true colour index to $+0^m.11$ and the mean distance modulus, after the corresponding correction of absolute magnitudes, to 11·4. This corresponds to

$$r = 1490 \pm 200 \text{ parsecs}, \quad d = 3'.7 = 1.6 \text{ parsecs}.$$

Collinder's corresponding values are 5200, 4'·0, 6·0.

The number of cluster stars, after correction for the background stars, is 18 against 19 got by counting. The magnitude distribution is given in the following table:—

m_{pv}	12	13	14	> 14
Number of stars . .	2	9	6	1

The three brightest stars are not cluster members.

NGC 366

R.A. = $1^h\ 00^m.2$, Dec. = $+61°\ 42'$ (1900.0)

This cluster is mentioned in the catalogue of Collinder. It lies in the square-shaped appendix of Baker's * region A north-east of γ Cass (noted as heavily obscured), in a kind of empty sac surrounded by a very rich region. This sac is readily visible in Ross-Calvert's *Atlas of the Milky Way*, plate 26, about 1 inch N.E. of γ Cass. The cluster's position, as given in the catalogues, is wrong, and that given above is determined by the co-ordinates of its principal star, the double star ADS 906 = *h* 1070, as remeasured by van Biesbroeck.†

On account of the peculiar situation of the cluster, star counting was not done in the usual way. A comparison of the cluster's region, containing 21 stars, with some neighbouring regions of the same size gave 19 as the number of cluster stars; this corresponds, with a diameter of 2'·9, to a star density per square degree of 10, 700.

Table II

Co-ordinates, Magnitudes and Distance Moduli of 10 *Stars* (*NGC* 366)

Zero point = star 12 = ADS 906, unit = 1'·0, X = west, Y = north
Co-ordinates of the assumed cluster centre $X = +0.4$, $Y = +0.9$

A: Normal distribution. B: Giants included

Star	X	Y	m_{pg}	m_{pv}	$m-M$	Star	$m-M$
16 ‖	−0·3	+0·7	12·15	11·48	6·8	3	10·2
17 ¶	−0·4	+0·7	12·49	11·81	7·0	18	10·5
9	+0·3	+2·0	11·87	11·43	7·8	15	10·7
12 ‡	0·0	0·0	11·59	11·23	8·2	12	11·2
3	+1·1	+0·5	13·69	13·32	10·2	9	11·4
18	−0·5	+0·3	13·65	13·32	10·5	16	11·5
15 §	−0·2	0·0	13·12	12·89	10·7	17	11·8
1	+1·4	+1·7	14·04	13·86	12·0	1	12·0
21	−1·0	+0·6	13·74	13·62	12·2	21	12·2
11	+0·1	+0·3	14·04	13·93	12·5	11	12·5

* *H.C.*, 424, 1939.
† *An. O. Belg.*, 13, 519, 1914.
‡ = ADS 906 = *h* 1070. § = C. ‖ = D. ¶ = E.
(F = star 19, not shown in this table because its image is distorted)

Table II gives the distribution of $m - M$. ADS 906 is not resolved on the plates and appears as No. 12. The sum of the magnitudes of its components ($11^m.5$, $12^m.4$) is $11^m.1$ and corresponds well with the measured magnitude, $11^m.2$. There was no comparison value for m_{pg} and the colour index seems therefore to be uncertain. The inclusion of this star in the usual treatment is thus to be regarded as provisional.

The table is given in two parts. The first part shows that we can regard the stars as being separated in two ranges, the first of which comprises 4 bright stars with higher colour indices. If we treat these 4 stars as giants (as was done in similar previous cases) we can rearrange the table and get the second part of Table II. We see that the giants fit well into the dwarf range.

There is still the question of absorption to be considered. The visual impression of the surroundings and Baker's results make it probable that the cluster region is subjected to absorption. The mean colour index of the main sequence stars ($+0^m.22$) suggests $0^m.20$ as possible colour excess. The mean distance modulus, under this assumption, is 12·44 (11·40 without absorption), corresponding to

$$r = 2160\ (1900) \pm 280 \text{ parsecs}, \quad d = 2'.9 = 1.8\ (1.6) \text{ parsecs}.$$

Collinder's corresponding values are 8000, 3'·0, 7·0.

The number of cluster stars is 19 (coinciding with the number got by counting), among them 4 giants. The magnitude distribution is given in the following table:—

m_{pv}	11	12	13	14
Number of stars	3	1	3	12

Finally it may be mentioned that there is a remarkably red star near the cluster; its position is $X = +6.7$, $Y = -5.0$; $m_{pg} = 13.7$, $m_{pv} = 11.3$ and the CI $= +2^m.4$.

NGC 381

R.A. $= 1^h\ 02^m.1$, Dec. $= +61°\ 3'$ (1900·0)

This cluster is mentioned in the catalogues of Trumpler and Collinder.

Direct counting gave 71 stars as the total number within the region, The distribution of stars within the cluster region and its surroundings results in a background density of 2960 stars per square degree and 43 cluster stars within a cluster region of 6'·7 diameter. Fig. 1 shows the star densities per square degree within the cluster region.

Table III gives the distribution of $m - M$. The mean distance modulus of the cluster group is 12·75, corresponding to

$$r = 3550 \pm 470 \text{ parsecs}, \quad d = 6'.7 = 7 \text{ parsecs}.$$

The results of previous investigations are

	Distance Parsecs	Diameter	
		′	Parsecs
Trumpler	2210	7·0	4·5
Collinder	2850	7·0	5·8

The number of cluster stars, after correction for the background stars, is 46 against 43 got by counting; the dispersion $\sigma = \pm 7$. The magnitude distribution is given in the following table:—

m_{pv}	11	12	13	14	> 14·5
Number of stars	2	5	18	19	2

NGC 433

R.A. $= 1^h\ 09^m.0$, Dec. $= +59°\ 36'$ (1900·0)

This cluster has, till now, received no attention.

Direct counting gave 18 stars as the total number within the region. The distribution

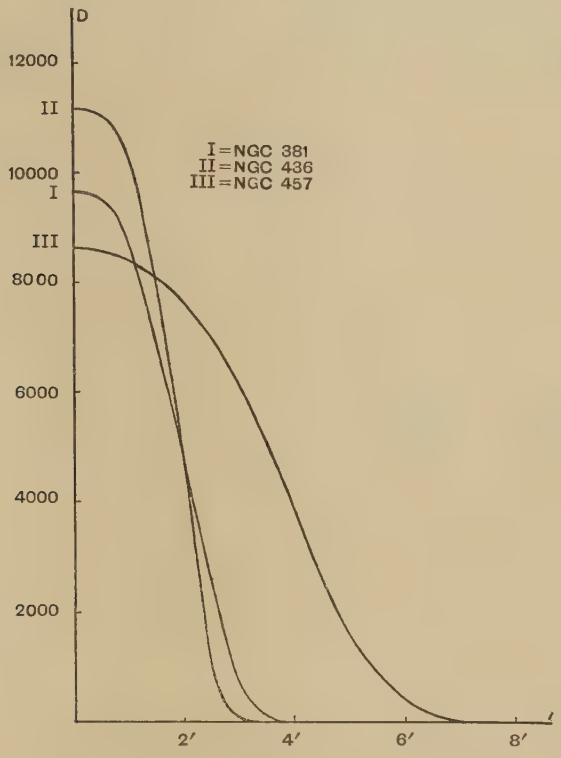

FIG. 1.—*Star density per square degree above background density.*

TABLE III

Co-ordinates, Magnitudes and Distance Moduli of 42 Stars (NGC 381)

Zero point = HD 6755, unit = 1′·0, X = west, Y = north
Co-ordinates of the assumed cluster centre $X = +9·6$, $Y = +2·2$

Star	X	Y	m_{pg}	m_{pv}	$m-M$	Star	X	Y	m_{pg}	m_{pv}	$m-M$
34	+ 9·4	+2·9	10·54	10·51	9·7	41	+ 9·2	+2·9	13·19	13·21	12·7
68	+ 7·0	+1·5	11·37	11·34	10·5	24	+10·1	+2·1	13·57	13·56	12·9
42	+ 9·1	−0·6	12·61	12·46	10·8	51	+ 8·4	−0·3	12·92	12·99	12·9
62	+ 7·5	−0·2	13·06	12·85	10·8	39	+ 9·2	+1·2	13·09	13·16	13·1
63	+ 7·4	+2·7	13·03	12·85	11·0	20	+10·8	+2·7	13·19	13·26	13·2
65	+ 7·4	+2·0	13·46	13·23	11·0	46	+ 8·8	+1·9	12·80	12·92	13·3
8	+11·8	+1·3	11·03	11·11	11·1	66	+ 7·3	+1·3	12·92	13·04	13·3
38	+ 9·2	+5·4	11·03	11·11	11·1	57	+ 7·9	−0·3	13·06	13·16	13·4
61	+ 7·5	−0·4	11·93	11·97	11·6	44	+ 9·0	+2·2	13·19	13·32	13·7
40	+ 9·1	+1·6	12·09	12·12	11·7	17	+11·0	+3·9	13·74	13·82	13·8
19	+10·8	+2·0	12·32	12·33	11·8	52	+ 8·2	+0·6	12·68	12·86	13·8
47	+ 8·7	+5·0	12·13	12·17	11·8	27	+10·0	−0·5	13·05	13·21	13·9
14	+11·2	−0·5	13·62	13·49	12·0	48	+ 8·7	+0·7	13·74	13·82	13·9
1	+12·7	+1·8	13·46	13·37	12·1	18	+11·0	−0·1	13·14	13·32	14·2
29	+ 9·9	+0·2	13·67	13·55	12·1	31	+ 9·7	+4·1	13·97	14·06	14·2
55	+ 8·0	−0·3	12·92	12·89	12·1	6	+12·1	+3·7	13·64	13·82	14·7
64	+ 7·4	+2·5	12·88	12·86	12·2	2	+12·5	+3·0	13·02	13·27	15·1
7	+12·0	+4·1	12·77	12·79	12·4	22	+10·2	+1·5	13·08	13·36	15·6
53	+ 8·0	+3·6	12·75	12·79	12·5	28	+ 9·9	+0·1	12·81	13·17	17·0
11	+11·4	+2·8	13·52	13·50	12·7	43	+ 9·0	+0·6	13·39	13·84	17·8
23	+10·2	+3·8	13·03	13·05	12·7	26	+10·1	+5·4	13·61	14·18	18·2

of stars within the cluster region and its surroundings results in a background density of 1830 stars per square degree and 13 cluster stars within a cluster region of 3′·4 diameter.

TABLE IV

Co-ordinates, Magnitudes and Distance Moduli of 15 Stars (NGC 433)

Zero point = HD 7332, unit = 1′·0, X = east, Y = north
Co-ordinates of the assumed cluster centre $X = +2{\cdot}9$, $Y = +8{\cdot}0$

Star	X	Y	m_{pg}	m_{pv}	$m-M$	Star	X	Y	m_{pg}	m_{pv}	$m-M$
10	+3·0	+8·1	10·10	8·76	1·5	1	+1·9	+8·0	13·46	13·55	13·7
16	+3·9	+7·4	13·87	13·72	12·0	3	+2·0	+8·0	13·87	13·93	13·7
5	+2·4	+9·0	13·74	13·63	12·2	7	+2·7	+7·5	12·86	13·03	13·7
9	+2·9	+7·7	12·97	13·00	12·6	18	+4·2	+7·3	14·04	14·11	14·0
13	+3·3	+7·6	12·87	12·93	12·7	17	+4·0	+9·3	13·41	13·55	14·1
14	+3·4	+8·4	12·87	12·93	12·7	8	+2·8	+9·1	13·35	13·46	14·3
15	+3·5	+7·3	14·04	13·95	12·7	11	+3·1	+7·0	13·62	13·76	14·3
12	+3·2	+6·7	13·96	13·93	13·1						

Table IV gives the distribution of $m-M$. The mean distance modulus of the cluster group is 13·27, corresponding to

$$r = 4500 \pm 700 \text{ parsecs}, \quad d = 3'{\cdot}4 = 4{\cdot}5 \text{ parsecs}.$$

The number of cluster stars, after correction for the background stars, is 12 against 13 got by counting. The magnitude distribution is given in the following table:—

m_{pv}	13	14
Number of stars	3	9

NGC 436

R.A. = $1^h\ 09^m{\cdot}4$, Dec. = +58° 17′ (1900·0)

This cluster is mentioned in the catalogues of Shapley, Trumpler and Collinder, and investigated by Trumpler and Raab.*

Direct counting gave 57 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 2000 stars per square degree and 41 cluster stars within a cluster region of 6′·0 diameter. Fig. 1 shows the star densities per square degree within the cluster region.

Table V gives the distribution of $m-M$. The mean colour index of the cluster group is $-0^m{\cdot}02$, while the spectral types B5 to B8, given by Trumpler, indicate a colour index of $-0^m{\cdot}12$. This would correspond to a colour excess of $+0^m{\cdot}10$. An excess of this value is also suggested by its presence in the neighbouring cluster NGC 457 (the next following in this paper). The new value of the range caused by assuming this colour excess agrees with the range expected according to A II, but it involves the inclusion of star number 43 (just outside the range in the table) into the cluster membership. The mean distance modulus is thus given by 13·56, corresponding to

$$r = 3120 \pm 320 \text{ parsecs}, \quad d = 6'{\cdot}0 = 5{\cdot}4 \text{ parsecs}.$$

* *Med. Lund*, Ser. II, 28, 1922.

TABLE V

Co-ordinates, Magnitudes and Distance Moduli of 30 Stars (NGC 436)

Zero point = BD 58° 209, unit = 1′·0, X = west, Y = south
Co-ordinates of the assumed cluster centre $X = +8.6$, $Y = +2.9$

Star	X	Y	m_{pg}	m_{pv}	$m-M$	Star	X	Y	m_{pg}	m_{pv}	$m-M$
25	+ 8·7	+3·9	12·17	11·82	8·8	26	+ 8·7	+2·6	11·00	11·26	13·3
30	+ 8·4	+3·0	11·37	11·34	10·5	7	+10·2	+2·2	12·92	13·07	13·7
49	+ 7·4	+2·1	13·86	13·54	10·7	19	+ 9·1	0·0	13·75	13·82	13·7
13	+ 9·5	+3·8	12·09	12·09	11·5	33	+ 8·2	+3·7	13·91	13·95	13·7
23	+ 8·8	+3·6	13·12	13·00	11·5	1	+11·0	+2·9	12·49	12·69	13·8
8	+10·2	+4·7	13·99	13·81	11·9	34	+ 8·2	+1·8	13·69	13·79	14·0
44	+ 7·7	+3·7	12·40	12·44	12·0	22	+ 8·9	+3·7	13·57	13·71	14·2
40	+ 7·9	+5·0	12·24	12·30	12·1	43	+ 7·7	+5·3	10·75	11·12	14·9
15	+ 9·5	+3·0	13·40	13·33	12·2	9	+ 9·9	+4·5	13·51	13·74	15·2
32	+ 8·3	+1·3	12·09	12·17	12·2	10	+ 9·8	+0·8	13·44	13·67	15·2
2	+10·9	+4·4	13·36	13·30	12·3	42	+ 7·7	+0·1	13·64	13·89	15·7
14	+ 9·5	+4·7	13·86	13·78	12·6	52	+ 6·5	+3·6	13·02	13·32	15·8
46	+ 7·5	+4·2	12·49	12·58	12·7	53	+ 6·3	+2·6	13·35	13·67	16·4
4	+10·7	+4·9	13·01	13·07	12·9	11	+ 9·7	+3·1	13·30	13·75	17·8
39	+ 7·9	+2·1	13·08	13·13	12·9	20	+ 9·1	+1·2	13·39	13·78	17·8

The results of the previous investigations are

	Distance Parsecs	Diameter	
		′	Parsecs
Shapley	720–1150	4·0	1·3
Trumpler	1970	6·0	3·5
Raab	...	6·0	...
Collinder	3190	5·0	4·6

The number of cluster stars, after correction for the background stars, is 38 against 41 got by counting; the dispersion $\sigma = \pm 6$. The magnitude distribution is given in the following table:—

m_{pv}	11	12	13	14	> 14
Number of stars	3	4	8	17	6

NGC 457

R.A. = 1^h 13^m·3, Dec. = +57° 45′ (1900·0)

This cluster is mentioned in the catalogues of Shapley, Trumpler and Collinder, and investigated by Trumpler, Raab, Hayford*, Anger Rieke†, Miller‡, and Klauder and Siedentopf.§ The co-ordinates, given by the catalogues, have here been corrected.

Direct star counting gave 192 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 1800 stars per square degree and 122 cluster stars within a cluster region of 13′·3 diameter. Fig. 1 shows the star densities per square degree within the cluster region. This density distribution agrees well with that given by Klauder and Siedentopf.

* *L.O.B.*, 16, 64, 1932.
† *H.C.*, 397, 1935.
‡ *H.B.*, 883, 30, 1931.
§ *A.N.*, 268, 197, 1939.

TABLE VI

Co-ordinates, Magnitudes and Distance Moduli of 94 Stars (NGC 457)

Zero point = star 172 = HD 7927, unit = 1′·0, X = west, Y = north
Co-ordinates of the assumed cluster centre $X = +4{\cdot}1$, $Y = +2{\cdot}8$

Star	X	Y	m_{pg}	m_{pv}	$m-M$	Star	X	Y	m_{pg}	m_{pv}	$m-M$
137*	+ 1·9	+4·4	10·45	8·45	±0	101	+ 3·8	−1·0	13·01	13·02	12·5
172†	0·0	0·0	5·70	5·19	1·3	112	+ 3·3	−1·0	12·92	12·95	12·6
82‡	+ 4·6	+1·5	10·31	9·52	4·2	155	+ 0·7	+7·3	13·56	13·53	12·7
105	+ 3·6	−2·3	13·69	12·24	4·5	3	+10·4	+2·7	12·27	12·39	12·8
94	+ 4·2	+1·3	9·19	8·78	5·5	83	+ 4·5	+7·7	12·81	12·88	12·8
179§	− 1·0	+4·8	10·90	10·30	5·9	106	+ 3·5	+2·6	13·25	13·26	12·8
92‖	+ 4·3	+5·2	10·00	9·57	6·2	171	0·0	+1·3	12·20	12·34	12·8
28	+ 7·7	+3·1	10·68	10·18	6·4	154	+ 0·8	+4·8	13·31	13·34	12·9
141¶	+ 1·7	−1·5	7·14	7·14	6·5	188	− 1·8	0·0	13·35	13·37	13·0
75**	+ 5·1	+3·1	10·17	9·77	6·6	42	+ 6·7	−0·8	11·09	11·10	10·6
78	+ 4·9	+3·8	11·19	10·68	6·8	72	+ 5·3	+1·6	13·19	12·95	10·8
191	− 2·3	+5·0	9·70	9·48	7·4	120	+ 2·9	0·0	13·13	12·91	10·8
113	+ 3·2	+6·5	11·74	11·34	8·1	40	+ 6·8	+5·8	11·00	11·07	11·0
85	+ 4·5	+3·2	10·42	10·24	8·3	156	+ 0·6	+3·5	12·43	12·34	11·0
119	+ 3·0	+0·7	9·97	9·75	8·4	144	+ 1·4	+2·4	12·62	12·55	11·5
23	+ 8·2	+5·3	12·07	11·67	8·5	57	+ 5·9	+2·1	13·50	13·34	11·6
30	+ 7·6	−0·1	9·33	9·32	8·6	168	+ 0·1	+7·3	12·36	12·35	11·7
27	+ 7·8	+6·7	9·62	9·59	8·8	69	+ 5·5	−1·5	13·67	13·51	11·8
99	+ 3·9	+2·1	13·70	13·18	9·3	21	+ 8·4	+3·4	12·04	12·10	11·9
62	+ 5·7	+0·9	12·72	12·37	9·4	190	− 2·1	+2·9	13·31	13·21	11·9
170	0·0	+1·8	13·68	13·19	9·4	56	+ 6·0	+2·5	13·06	13·00	12·0
15	+ 8·7	+4·9	11·94	11·71	9·5	58	+ 5·9	+4·1	9·33	9·63	12·1
54	+ 6·0	+4·4	9·78	9·82	9·5	159	+ 0·4	+1·1	12·23	12·28	12·1
84	+ 4·5	+8·5	11·31	11·15	9·5	129	+ 2·4	−1·6	10·63	10·85	12·3
146	+ 1·2	+2·0	11·89	11·70	9·8	163	+ 0·3	+6·0	13·39	13·33	12·3
103	+ 3·6	+5·9	12·49	12·30	10·4	134	+ 2·1	+1·9	12·36	12·44	12·4
90	+ 4·3	+1·9	11·18	11·16	10·5	143	+ 1·3	+7·3	13·82	13·72	12·4
181	− 1·2	+5·2	13·82	13·47	10·5	87	+ 4·5	−0·1	13·30	13·26	12·5
24	+ 8·2	+1·4	13·96	13·94	13·1	86	+ 4·5	+2·5	13·69	13·82	14·2
73	+ 5·3	+2·0	12·67	12·78	13·1	97	+ 4·0	+7·1	13·87	13·99	14·4
158	+ 0·4	+1·5	13·39	13·43	13·1	177	− 0·8	−1·8	13·05	13·27	14·7
100	+ 3·9	+0·9	12·36	12·52	13·2	25	+ 8·1	+4·0	13·55	13·75	14·9
116	+ 3·1	+2·1	13·25	13·32	13·2	187	− 1·8	+1·6	13·34	13·57	15·2
68	+ 5·4	+3·9	12·26	12·44	13·3	53	+ 6·1	+5·1	13·05	13·31	15·3
174	− 0·2	+5·1	13·67	13·70	13·3	31	+ 7·3	+1·7	12·61	12·93	15·7
43	+ 6·6	+4·6	13·14	13·24	13·4	79	+ 4·9	−0·3	13·49	13·75	15·8
178	− 0·8	+2·3	12·91	13·04	13·4	29	+ 7·6	+6·2	11·98	12·37	16·4
65	+ 5·7	−0·9	13·91	13·93	13·5	36	+ 6·9	+3·9	13·60	13·91	16·6
74	+ 5·2	−2·9	13·11	13·22	13·5	118	+ 3·0	+7·3	12·26	12·68	16·7
33	+ 7·2	+1·2	13·56	13·64	13·6	4	+ 9·8	+2·2	13·90	14·22	17·0
45	+ 6·5	+3·4	13·19	13·31	13·7	5	+ 9·8	+0·2	12·58	12·98	17·0
80	+ 4·9	−1·5	13·87	13·93	13·7	46	+ 6·5	+2·4	12·86	13·25	17·3
192	− 2·6	+2·8	13·87	13·92	13·7	77	+ 4·9	+5·6	13·18	13·56	17·6
167	+ 0·1	+1·5	13·35	13·46	13·8	127	+ 2·4	+4·4	13·19	13·57	17·6
1	+10·5	+4·2	13·20	13·35	14·0	63	+ 5·7	+1·8	13·90	14·25	17·7
110	+ 3·4	+2·0	13·91	13·99	14·0	98	+ 4·0	+0·7	13·19	13·68	17·7
149	+ 1·0	+4·9	13·05	13·22	14·0	126	+ 2·5	+1·1	13·55	13·94	17·9

* = BD 57° 258 = AG 1127. † = ϕ Cass = ADS 1073 = AG 1130. ‡ = BD 57° 251 = ADS 1069.
§ = BD 57° 261 = Mlb. 153. ‖ = BD 57° 253 = ADS 1063. ¶ = HD 7902 = AG 1126. ** = Es 407.

Table VI gives the distribution of $m - M$. The mean distance modulus of the cluster group is 11·88, corresponding to $r = 2380$ parsecs. A comparison of the mean colour index of $+0^m.02$ with that indicated by the spectral types (Trumpler's catalogue notes 29 stars ranging between B2 and A0) suggests, however, a colour excess of about $+0^m.10$. This is further supported by the $(m - M)$ range of 6·3, which is more than should be expected and corresponds better to the corrected colour index of $-0^m.08$. After the redetermination of $m - M$ on this assumption, the mean is 12·70, corresponding to

$$r = 2370 \pm 300 \text{ parsecs}, \quad d = 13'.3 = 9.2 \text{ parsecs}.$$

The close coincidence of this distance with that determined without assuming absorption is due to the accidental compensation of the absorption effect on the apparent magnitude by the correction applied to the absolute magnitudes on account of colour excess (form. (1) in A II).

The results of the previous investigations are

	Distance Parsecs	Diameter	
		′	Parsecs
Shapley	420–660	10·0	1·9
Trumpler	1480	12·0	5·2
Raab	1030	17–20	5·6
Collinder	1520	14·5	6·4
Anger Rieke	2000	...	...

Anger Rieke uses three different values of the absorption coefficient; the one on which the above distance is based is $0^m.4$, the nearest value to that used here ($0^m.35$).

The number of cluster stars, after correction for background stars, is 102. We find, however, among the foreground stars 5 double or multiple star systems. This is quite out of proportion in density against the average of 2 double star systems per square degree (the area of the cluster region is 0·04 square degree) and suggests that most of these systems are really members of the cluster. The reason for the small apparent distance modulus may be explained by the fact that the measured magnitudes are (with one exception) total magnitudes of the systems. It can easily be shown that the difference between the photographic and photovisual total magnitudes gives only a fictitious value of the colour index. This value has the effect of making $m - M$ for the system smaller than the true value. Hence the true distance is larger than that determined by observation. The comparatively high colour indices suggest that the primary stars might be giants, later than F0, and in this case all 4 systems would fit in the range of the cluster group; but no giants are recorded in Trumpler's spectral data. We find, however, the primary of one of the systems, No. 82 = Es 408, given as of spectral type B3 in Hayford's paper (star number 5). The application of this spectral type shifts the distance modulus $m - M$ within the range. All the cluster stars measured by Trumpler are of early spectral types (B2 to A0), so that it seems reasonable to adopt accordingly early spectral types for the primaries of the other 3 systems. We are thus able to bring them within the limits of the cluster group. Since the absolute magnitudes of the secondary star in a main sequence binary system must be fainter than that of the primary, it may be assumed that in general it must be of later spectral type, as found by Doig.* A thorough spectral investigation of these systems, which unfortunately cannot be done at this observatory, might largely contribute to our knowledge on the relation between galactic clusters and multiple stars.

The exception mentioned above is No. 172 = ϕ Cass. The companion of this star, B = No. 162, shows a distorted image on the photographic plate and therefore does not appear in Table VI. (It may be mentioned here that among the relations of three other stars to one another and to ϕ Cass in ADS 1073, namely C = 141, D = 119, E = 129, the

* *M.N.*, 82, 372, 1922.

D in the pair CD ought to be E. The mistake is due to copying this pair from Espin's catalogue *, where his D is equivalent to E of ADS. This will be clear by looking at the data of CE, which are nearly identical with those of the alleged CD.) ϕ Cass belongs certainly to the foreground stars and this is also confirmed by the spectral parallax of $+0''.002$.†

The applied correction (3 double stars and 1 quadruple star) enlarges the star number to 112, against 128 got by counting (after a similar correction); the dispersion $\sigma = \pm 11$. The difference of 16 is larger than the dispersion would indicate and it might be due either to an accidental star distribution, or to the cause suggested in the introduction, or to both of them.

The magnitude distribution is given in the following table:—

m_{pv}	9	10	11	12	13	14	> 14
Number of stars	1	9	8	11	26	45	12

Finally it may be noted that besides the very red star 137 (the first one in Table VI), which seems to be a very near neighbour of the Sun, there is another very red star near the cluster region: $X = +9.3$, $Y = -5.2$, $m_{pv} = 12.55$, no trace of which can be found on any available photographic plate; its colour index must therefore be greater than $+3^m.0$.

NGC 637

R.A. $= 1^h\ 36^m.0$, Dec. $= +63°\ 32'$ (1900·0)

This cluster is mentioned in the catalogues of Shapley, Trumpler and Collinder.

Direct counting gave 30 as the total star number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 2230 stars per square degree and 24 cluster stars within a cluster region of $3'.5$ diameter.

TABLE VII

Co-ordinates, Magnitudes and Distance Moduli of 23 Stars (NGC 637)

Zero point = HD 10304, unit = $1'.0$, X = east, Y = north

Co-ordinates of the assumed cluster centre $X = +1.6$, $Y = +9.3$

Star	X	Y	m_{pg}	m_{pv}	$m-M$	Star	X	Y	m_{pg}	m_{pv}	$m-M$
17	+ 1·8	+11·1	10·29	9·73	5·6	9	+ 0·9	+10·5	13·19	12·99	11·0
16	+ 1·6	+ 8·8	10·22	9·77	6·3	26	+ 2·3	+ 9·0	11·99	11·96	11·2
30	+ 2·7	+ 8·2	13·38	12·45	6·6	25	+ 2·2	+ 8·1	14·09	13·86	11·7
15	+ 1·6	+ 8·2	10·08	9·75	7·0	5	+ 0·6	+10·5	13·19	13·18	12·5
8	+ 0·9	+ 9·1	12·82	12·10	7·1	3	+ 0·6	+ 8·5	14·01	13·93	12·7
14	+ 1·6	+ 9·1	10·19	9·91	7·4	2	+ 0·5	+ 8·5	13·70	13·67	12·9
11	+ 1·2	+ 8·8	11·79	11·37	8·0	22	+ 2·2	+ 8·4	14·13	14·09	13·2
7	+ 0·7	+ 9·6	13·41	13·02	9·8	4	+ 0·6	+10·2	13·70	13·73	13·3
29	+ 2·6	+10·4	13·01	12·72	10·1	10	+ 1·0	+ 8·0	13·41	13·49	13·5
23	+ 2·1	+ 9·2	10·89	10·94	10·7	1	+ 0·4	+ 9·2	14·01	14·11	14·3
28	+ 2·6	+10·1	12·39	12·25	10·7	18	+ 1·9	+ 8·5	12·80	13·05	14·6
27	+ 2·6	+ 9·6	11·61	11·60	10·9						

Table VII gives the distribution of $m-M$. The mean distance modulus is 11·73, corresponding to

$$r = 2220 \pm 350 \text{ parsecs}, \quad d = 3'.5 = 2.2 \text{ parsecs}.$$

**M.N.*, 68, 205, 1908.

† Schlesinger, *Cat. of Par.*, 1935.

The results of the previous investigations are

	Distance Parsecs	Diameter	
		′	Parsecs
Shapley	870–1380	·3·0	1·2
Trumpler	3340	3·5	3·4
Collinder	3470	2·5	2·5

If we consider the four bright stars 17, 16, 15 and 14 as giants (a consideration justified by the narrow range of their $m-M$ and by their conspicuous density) they are then included among the cluster stars, their corresponding $m-M$ ranging between 9·7 and 9·9. The number of cluster stars, after correction for the background stars, is 20 (including 4 giants) against 24 got by counting; the dispersion $\sigma = \pm 5$. The magnitude distribution is given in the following table:—

m_{pv}	10	11	12	13	14
Number of stars . . .	4	1	1	7	7

NGC 743

R.A. $= 1^h\ 51^m{\cdot}9$, Dec. $= +59°\ 41'$ (1900·0)

This cluster is mentioned only in NGC. It consists of a coarse condensation of stars, most of which are brighter than 13^m. The boundaries of this condensation correspond to the edges of a triangle (already noted in Herschel's remarks). No condensation of fainter stars is recognizable. It was therefore not necessary to proceed in the usual way in counting the star number: a comparison of the cluster region with some regions of equal size in the neighbourhood gave 6 as number of background stars of the same magnitudes. This results, after subtraction from 16 (the total number), in 10 cluster stars within the cluster region of about 5′·3 diameter. (The value of the diameter is given only approximately on account of the particular shape of the cluster.)

TABLE VIII

Co-ordinates, Magnitudes and Distance Moduli of 15 Stars (NGC 743)

Zero point = star 1 = HD 11858, unit = 1′·0, X = east, Y = south
Co-ordinates of the assumed cluster centre $X = +2{\cdot}6$, $Y = +2{\cdot}6$

Star	X	Y	m_{pg}	m_{pv}	$m-M$	Star	X	Y	m_{pg}	m_{pv}	$m-M$
6	+ 2·0	+2·9	11·75	11·29	7·7	3	+ 1·2	+4·5	9·74	9·85	10·2
5	+ 1·7	+1·0	10·99	10·68	8·0	2	+ 1·1	+3·6	10·64	10·67	10·3
11	+ 3·3	+5·0	12·55	12·01	8·0	8	+ 2·1	+6·5	10·84	10·86	10·4
7	+ 2·0	+4·0	12·25	11·79	8·2	4	+ 1·2	+3·1	10·84	10·97	11·4
1	0·0	0·0	9·14	9·20	9·0	9	+ 2·6	+1·0	13·37	13·24	11·7
15	+ 5·1	+2·4	9·69	9·69	9·1	16	+ 5·1	+1·4	13·24	13·45	14·6
14	+ 4·1	+3·5	11·94	11·66	9·2	10	+ 2·7	+3·9	11·99	12·34	15·8
13*	+ 3·7	+4·6	9·95	10·00	9·8						

Table VIII gives the distribution of $m-M$. The mean distance modulus is 10·12, corresponding to

$$r = 1060 \pm 210 \text{ parsecs}, \quad d = 5'{\cdot}3 = 1{\cdot}6 \text{ parsecs}.$$

* Burnham 1020.

The number of cluster stars is 9 against 10 found by comparison. The magnitude distribution is given in the following table:—

m_{pv}	9	10	11	12	13
Number of stars	1	3	3	1	1

Star number 13 is H 1098 and the companion is No. 12 (not listed in Table VIII because its image is distorted).

Norman Lockyer Observatory,
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A PHOTOGRAPHIC SURVEY OF GALACTIC CLUSTERS: VI. THE CLUSTER GROUP IN CASSIOPEIA

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Summary

The constitution of the group of 19 clusters within the Cassiopeia region is investigated. It is found that there are two condensations of 6 clusters each, at about 1280 and 2170 parsecs. A table of all the relevant data is given and a relation between apparent space density and distance is shown graphically. The differences between background densities in different investigations are explained. Finally it is suggested that on account of the random distribution of colour excesses the obscuring clouds might be composed, at least partially, of small scattered clouds of various obscuring power.

In the previous paper the investigation of all the galactic clusters within the Cassiopeia region, outlined roughly by the 5 brightest stars, was completed as far as the clusters were accessible to the Mond Astrograph. It seems worth-while to review the results for this group of 19 clusters, especially as the region in which they are found has previously been specially studied from other points of view. Schalén * investigated the eastern part of this region taking into account stars down to $11^{m}.0$, and Vanäs † continued this investigation. Graff ‡ published colorimetric colour excesses of stars down to about $7^{m}.0$. Greenstein § uses this region in his consideration of the absorption coefficient and Bok || also refers to it in a similar connection. Baker ¶, eventually, carried a detailed star counting down to $15^{m}.0$ throughout a large region almost surrounding the present one. Two special regions, the surroundings of galactic clusters, were investigated by Klauder and Siedentopf ** who counted stars in one case down to $17^{m}.7$. Finally, the present region is precisely the same as that in which Trumpler †† found a group of clusters.

The table on p. 192 contains all the relevant data yielded by this survey. It is arranged in order of increasing distances. The classes referred to are defined in an earlier paper ‡‡ on the structure of galactic clusters (designated later A).

Although the relations established in that paper were based on spectral observations of Trumpler and others they remain the same when based on colour index observations, as in the present example. The class MC seems to be restricted to the smaller distances, due to the fact that clusters of this class consist of stars of lower luminosity and are

* *Ups. Med.*, 37, 1928; 55, 1931.
† *Ups. Med.*, 81, 1940.
‡ *Mit. O. Wien*, 1, 313, 1937; 1, 380, 1939.
§ *H.A.*, 50, 360, 1936.
|| *The Distribution of the Stars in Space*, 76, 1937.
¶ *H.C.*, 424, 1939.
** *A.N.*, 268, 197, 1939.
†† *L.O.B.*, 14, 184, 1930.
‡‡ *M.N.*, 102, 205, 1942.

Structural Qualities of 19 Clusters within the Cassiopeia Region

Col. 1 = Name; 2 = Survey no.; 3, 4 = galactic longitude and latitude; 5 = Class; 6 = Number of stars; 7 = Colour excess; 8 = Distance in parsecs; 9 = Diameter in parsecs: 10 = Volume in cubic parsecs; 11 = Number of stars per cubic parsec; 12 to 14 = Background densities according to Alter, Baker, Klauder and Siedentopf.

NGC	A *	l_g	b_g	Cl	n	CE	$r \pm$m.e.	d	V	D	A.	B.†	K. + S.
1	2	3	4	5	6	7	8	9	10	11	12	13	14
Anon.	III	88·4	+0·6	MC	51	...	375 ± 20	0·8	0·27	189·0	4000	C 370	...
146	III	88·5	+0·8	MC	20	...	750 35	1·5	1·8	11·1	4000	C 370	...
743	V	99·0	−1·0	MH	9	...	1060 210	1·6	2·2	4·1	...	...	...
129	III	88·0	−2·3	MC	28	...	1170 60	1·9	3·6	7·8	3500	D 750	...
663	II	97·2	−0·4	MH	161	·62	1200 100	5·7	96·4	1·7	2600	F 490	2100
225	III	89·7	−0·7	MH	17	·50	1275 50	4·7	54·5	0·3	...	...	...
189	V	89·2	−1·8	MH	20	·20	1490 200	1·6	2·15	9·3	2050	D 750	...
103	III	87·5	−1·1	MC	33	...	1500 70	2·4	7·25	4·55	4600	D 750	...
581	I	95·8	−1·3	MH	52	...	2040 180	4·5	47·6	1·1	3900	R_1 725	...
654	II	96·8	+0·1	MH	41	·60	2060 200	3·3	18·8	2·2	1600	F 490	...
Tr.1	II	96·0	−0·6	MH	30	...	2150 200	3·3	18·8	1·6	3250	F 490	...
366	V	92·4	0·0	GC	19	·20	2160 280	1·8	3·1	6·1	...	A 270	...
637	V	96·2	+2·2	GC	20	...	2220 350	2·2	5·6	3·6	2200	F 490	...
457	V	94·4	−4·0	MH	112	·10	2370 300	9·2	408·0	0·3	1800	Q_2 630	2400
559	II	94·9	+1·2	MH	39	...	2770 250	3·5	22·4	1·74	3000	A 270	...
659	II	97·1	−1·0	GH	36	...	3000 250	4·8	57·8	0·7	3300	F 490	...
436	V	93·8	−3·5	MH	40	·10	3120 320	5·4	80·0	0·5	2000	Q_1 550	...
381	V	92·7	−0·8	MH	45	...	3550 470	7·0	180·0	0·25	3000	A 270	...
433	V	93·7	−2·2	MH	13	...	4500 ± 700	4·5	47·6	0·27	1830	Q_1 550	...

therefore more easily recognised at shorter distances. The linear diameters (and therefore the volumes) seem to increase with distance. This also is not surprising, since the smaller clusters may not be recognized at larger distances. The spatial density seems to decrease with distance, due partly to the apparent increase in volume and partly to the loss of less luminous stars with increasing distances. These effects of selection are referred to in A.

The present data enable us to express the density relation better than in Table VI of A. In Fig. 1A the densities are plotted against distances and a curve is drawn to indicate the relation. A certain scatter is evidently to be expected, and if we note that the most divergent points represent clusters subjected to absorption (and so to more uncertainty in distance and star number) the curve might be considered as reasonably representing this relation. The high average spatial density of the anonymous cluster (189 stars per cubic parsec) corresponds to an average internal star distance of about 0·5 of a light-year. It probably gives us an idea of the true spatial star densities in galactic clusters which otherwise appear, by selection, much lower, as already pointed out in A.

The spatial distribution of the clusters is illustrated by the projection on the galactic plane, given in Fig. 1B. It will readily be seen that there are two compact groups of clusters, while the remaining clusters lie in the foreground and the background. The first group (of 6 clusters) lies between 1000 and 1500 parsecs, its mean distance is 1280 parsecs and the mean cluster distance from the group centre is 180 parsecs. The second group (also of 6 clusters) lies between 2000 and 2400 parsecs, its mean distance is 2170 parsecs and the mean cluster distance from the group centre is 120 parsecs. Trumpler's

* *M.N.*, 100, 387, 1940; 101, 89, 1941; 101, 298, 1941; 104, 179, 1944.

† The capitals give the areas of Baker's Fig. 1.

cluster group, mentioned above, is composed of 10 clusters (4 of which are investigated spectroscopically), its mean distance is 2070 parsecs, but the mean cluster distance from the group centre is about 250 parsecs because the group extends between 1480 and 2340 parsecs. The differences between the distances of this paper and those of Trumpler are due to a different method of determination and to the use of fainter stars.

The last three columns of the table give the background densities of stars per square degree. The noticeable difference between Baker's values, taken from his Table IV, and those of the survey is due partly to a different limiting magnitude and partly to different emulsions. Baker's limiting magnitude is 15, while the present is about 16;

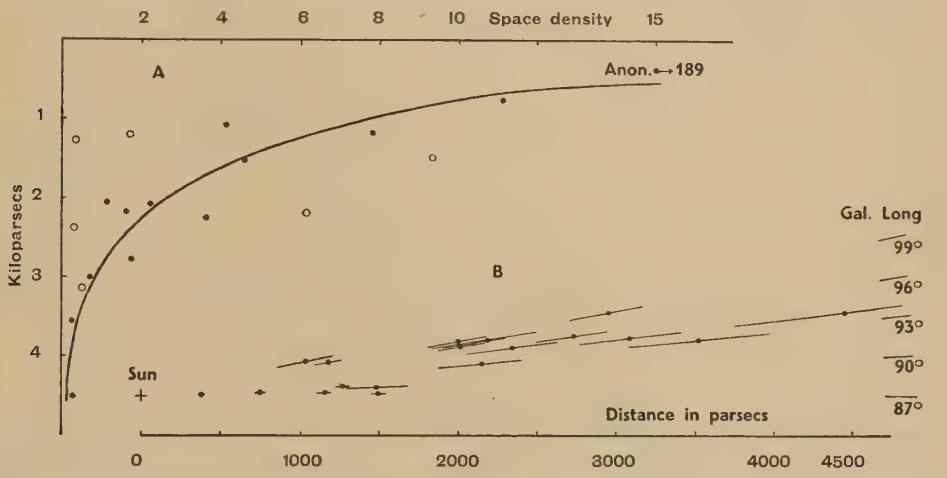

FIG. 1A.—*Relation between spatial density and distance. Open circles indicate obscured clusters.*

FIG. 1B.—*Projection of clusters on galactic plane. Straight lines indicate uncertainty of distance.*

Baker's plates are photographic (unable to register the numerous redder stars), those of the survey are photovisual.

The part played by the increase of the limiting magnitude is found by comparison of Klauder and Siedentopf's values (also from photographic plates) with Baker's. In both cases, NGC 663 and 457, the background density found by Klauder and Siedentopf is about four times that of Baker and their limiting magnitude is about 17. This corresponds to a doubling of density for 1^m increase in limiting magnitudes for this magnitude range. This is also confirmed by another counting of the NGC 663 region by Klauder and Siedentopf down to $17^m.7$, giving a density of 3550 against 2100 for 17^m.

The large remainder of the density surplus of the survey above the doubling (for about 1^m difference in the limiting magnitudes) is explained by the steady increase of the luminosity function towards the red stars, which are omitted on the photographic plates and recorded on the photovisual ones. This is also supported by considering the ratio of the two background densities Alter/Baker. In Baker's regions A and C this is about 11, and in all the other regions about 4·5 (±1·2) on the average. The regions A and C are subjected, according to Baker, to absorptions of $1^m.9$ and $1^m.5$, respectively, while the average absorption of the other regions concerned is only $0^m.7$ (±0·3).* The increased absorption in the regions A and C will affect the photographic plates much more than the photovisual ones, thus eventually increasing the ratio of densities (and confirming the large influence of the use of different emulsions).

The table shows no increase of the colour excess with the increase of distance (the

* *Note added in proof*:—New data published by Baker and Nantkes in *Ap. J.*, 99, 125, 1944, do not alter the ratios Alter/Baker appreciably, but the absorptions in the regions A and C are now $2^m.6$ and $1^m.5$ respectively, while the average absorption of the other regions is $1^m.0 \pm 0^m.4$.

opposite behaviour is accidental), and this indicates that there is not only no appreciable continuous absorbing layer but that the obscuring clouds are even more scattered than is shown in Baker's Fig. 1. We find that clusters lying within one of Baker's areas show not only different ratios (Alter/Baker) of background densities (this could be partly explained by irregular star distribution) but also different colour excesses. This suggests that the apparent obscuring clouds of comparatively larger size may be, at least partially, composed of a number of much smaller scattered clouds of various obscuring power.

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1944 *April* 19.

OBSERVATIONS OF EROS AT THE CAPE OBSERVATORY IN 1938 AND 1942

(Communicated by H.M. Astronomer)

(Received 1944 May 5)

Summary

The result of observations of Eros made photographically at the Royal Observatory at the Cape in the years 1938 and 1942 are given. Those for 1942 are mostly on the system of FK3 and are compared with an accurate ephemeris.

Observations of the planet Eros made near the times of its closest approaches to the Earth are used for direct determinations of the solar parallax by the diurnal method or by observations made at distant observatories. Those made at other times can be used for the determination of the solar parallax from the perturbations produced at the close approaches to the Earth. This effect increases with the time, but unfortunately errors in the calculations of the orbit may also increase with the time. If the observations are carefully reduced in a definite system of star places they can be used for determining systematic errors in the star places.

The opposition of 1937–38 was much more favourable for observation in the northern hemisphere, and no attempt was made at the Cape Observatory to secure an extended series of observations. In the opposition of 1940 the astrographic telescope was dismantled for use at the total eclipse of October 1 of that year, but in 1942, especially in view of the probable scarcity of observations made elsewhere, an effort was made to secure as many observations as possible.

In 1938 all the observations were made with the Astrographic telescope west of the pier. In 1942 generally the first two or three exposures were made with the telescope west of the pier before Eros reached the meridian, then observations were made with the Victoria telescope near the meridian, and finally the Astrographic telescope was reversed and several exposures made with the telescope east of the pier. As the same plate was used for the two sets of exposures with the Astrographic telescope, the mean of the two sets should eliminate any displacement due to tilt of the plate.

The plates were reduced with the most suitable comparison stars for which star places were available. In general, especially in 1942, the planet was much fainter than the comparison stars. The plates for 1938 April 1 and 2 and for 1942 August 8 to October 5 were reduced with star places kindly supplied before publication by Professor Brouwer of Yale Observatory. These reductions are on the FK3 system. Notes at the end of the observations give the star places used for the other observations. For the plates taken with the Victoria telescope the reductions were made by means of intermediary stars the positions of which were determined on the Astrographic plates.

For 1938 no accurate ephemeris of Eros was available and the observed positions only are given. These have been corrected for parallax and refer to the equinox of 1938·0. To compare with an ephemeris giving actual positions referred to the equinox 1938·0 aberration has to be applied to the positions given and the times of observation have to be antedated by the light-time.

For 1942 an ephemeris by Stracke * was available. The positions given have been corrected for parallax and aberration. For comparison with the ephemeris it has only been necessary to subtract the light-time from the time of observation. It should be noted that the quantity tabulated by Stracke as "Fixstern-Aberr." is not the aberration but includes precession and nutation.

In 1938 the plates were taken by Dr. Stoy, in 1942 generally by Dr. Jackson. All the plates were measured by Mr. Peirce, and he was assisted in the reductions by Mrs. Andrews and Mr. Menzies.

In the tables giving the results of the observations the number of exposures combined to a mean is given, while the letters W and E indicate that the Astrographic telescope was used west or east of the pier, and the letter V indicates the Victoria telescope, always west of the pier.

U.T.	R.A.	Dec.	No.
	h m s	° ′ ″	
1938 Jan. 31·79823	4 59 59·41	+24 44 20·3	1
Feb. 9·79000	5 25 37·43	+18 03 39·1	2
18·77922	5 52 01·11	+12 22 42·8	3
21·80061	6 00 56·42	+10 42 10·4	3
23·78315	6 06 48·35	+ 9 39 57·3	3
Mar. 28·76734	7 44 06·54	− 1 51 15·5	3
Apr. 1·76087	7 55 45·56	− 2 43 46·8	4
2·76232	7 58 40·65	− 2 56 18·2	4

U.T.	R.A.	Dec.	O − C		No.	
	h m s	° ′ ″	s	″		
1942 June 28·13462	22 07 01·66	−12 15 45·6	−0·01	−0·1	1	E
July 22·92459	21 47 26·11	−10 10 17·3	+ ·02	+1·2	3	W
·92998	25·68	15·6	+ ·05	+1·8	3	W
24·01797	21 45 53·88	−10 06 32·6	+ ·09	+1·3	3	W
·04638	51·40	26·6	+ ·06	+1·6	2	E
Aug. 8·92788	21 18 59·98	− 9 27 18·9	+ ·06	+0·4	3	W
·94232	58·39	17·7	+ ·08	+0·1	4	V
·94943	57·59	16·9	+ ·08	+0·2	3	V
·96254	56·14	16·0	+ ·09	−0·2	3	E
9·92629	21 17 08·23	− 9 25 41·2	+ ·05	0·0	3	W
·93718	07·03	39·9	+ ·07	+0·3	3	V
·94376	06·30	39·2	+ ·08	+0·4	3	V
·95666	04·85	37·8	+ ·08	+0·6	4	E
13·90864	21 09 41·39	− 9 19 54·8	+ ·04	+0·3	4	W
·93000	39·04	53·1	+ ·08	+0·2	3	E
14·92446	21 07 48·02	− 9 18 37·1	+ ·04	−0·1	3	W
·93284	47·09	36·4	+ ·04	0·0	3	V
·93802	46·52	36·0	+ ·04	0·0	3	V
·95977	44·10	34·6	+ ·04	−0·2	4	E
18·90333	21 00 31·70	− 9 14 04·5	+ ·10	0·0	4	W
·91705	30·21	03·7	+ ·09	−0·1	3	V
·92417	29·45	03·4	+ ·09	−0·2	3	V
·93952	27·79	02·4	+ ·08	−0·1	4	E
19·90359	20 58 44·74	− 9 13 03·5	+ ·06	−0·5	4	V
·90606	44·48	03·1	+ ·06	−0·2	3	W
·91125	43·92	03·1	+ ·05	−0·6	4	V
·91895	43·12	02·6	+ ·06	−0·5	3	E
21·90274	20 55 15·50	− 9 11 07·6	+ ·04	−0·2	3	V
·90426	15·34	07·8	+ ·04	−0·4	3	W
·90858	14·91	07·3	+ ·05	−0·2	3	V
·92285	13·44	06·4	+ ·04	0·0	3	E
30·87136	20 41 15·53	− 9 03 46·1	+ ·04	+0·2	3	V
·87661	15·10	45·8	+ ·04	+0·3	3	V
·89453	13·62	45·0	+ ·04	+0·3	3	E

* *A.N.*, 272, 27, 1941.

U.T.	R.A.	Dec.	O − C		No.	
	h m s	° ′ ″	s	″		
Aug. 31·85689	20 39 55·24	− 9 03 01·9	+ ·05	+0·2	3	W
·86640	54·46	01·5	+ ·03	+0·2	3	V
·87228	53·98	01·2	+ ·02	+0·3	4	V
·88396	53·03	00·7	+ ·01	+0·2	4	E
Sept. 3·84856	20 36 08·34	− 9 00 47·8	+ ·08	+0·8	2	W
·85830	07·62	47·6	+ ·06	+0·6	3	V
·86479	07·15	47·4	+ ·05	+0·4	3	V
·88146	05·96	46·6	+ ·05	+0·5	2	E
5·84336	20 33 51·74	− 8 59 18·2	+ ·05	+0·3	3	W
·85370	51·06	17·8	+ ·04	+0·2	3	V
·86119	50·58	17·4	+ ·05	+0·3	3	V
·87685	49·55	17·1	+ ·04	−0·2	3	E
6·82719	20 32 48·89	− 8 58 32·8	+ ·05	+0·3	3	W
·84305	47·90	32·0	+ ·05	+0·4	3	V
·85059	47·42	31·8	+ ·04	+0·3	3	V
·86804	46·34	31·2	+ ·04	+0·2	2	E
7·81807	20 31 48·64	− 8 57 46·3	+ ·04	+0·4	3	W
·83570	47·59	45·5	+ ·03	+0·4	3	V
·84322	47·14	44·9	+ ·03	+0·7	3	V
·85700	46·34	44·5	+ ·04	+0·4	3	E
13·80796	20 26 51·46	− 8 52 38·9	+ ·04	+0·2	3	W
·82062	50·89	37·8	− ·03	+0·6	3	V
·82800	50·61	37·6	− ·02	+0·3	2	V
·84451	50·02	36·7	+ ·05	+0·3	3	E
15·81740	20 25 37·85	− 8 50 41·0	+ ·03	+0·4	2	V
17·83025	20 24 37·20	− 8 48 33·6	+ ·02	+0·2	1	W
Oct. 1·76070	20 23 17·80	− 8 28 00·2	− ·02	+0·2	2	W
·78318	18·16	− 8 27 57·4	+ ·01	+0·4	2	E
2·76664	20 23 33·56	− 8 26 02·0	− ·02	+0·4	2	W
·78730	33·92	25 59·8	− ·01	+0·1	2	E
4·76854	20 24 12·90	− 8 21 53·7	+ ·02	+0·2	3	V
5·75957	20 24 36·20	− 8 19 43·3	+0·01	+0·5	3	V

Notes on Comparison Stars used

An asterisk indicates that proper motion has been applied.

1938 Jan. 31. Yale, Vol. 10.
Feb. 9. d'Abbadia (1900) 2537*, 44, 49*, 62*, 65*, 66*, 85*, 2594*.
18. A.G. (1875) 1865*, 66*, 68, 72*, 77*, 89, 98*, 1899.
21. A.G. (1875) 1960*, 77, 89, 1996.
23. A.G. (1875) 2667, 68, 70, 75, 85, 2689.
Mar. 28. Yale, Vol. 5.
Apr. 1–2. Yale, unpublished. FK3 system.
1942 June 28, July 24. Yale, Vol. 11. Eichelberger's system.
Aug. 8–Oct. 5. Yale, unpublished. FK3 system.

Erratum

M.N., **103**, No. 5, H. Zanstra's paper:

Page 273, equation (4*a*), *for* $\pi r_{\text{rad}}\omega$ *read* $\frac{\pi r_{\text{rad}}\omega}{\lambda}$.